Supersymmetric Compactification and Higher-Dimensional Rearrangement of Photonic Lattices

Tom A. W. Wolterink*, **Matthias Heinrich, and Alexander Szameit**

Institute for Physics, University of Rostock, Albert-Einstein-Straße 23, 18059 Rostock, Germany
* tom.wolterink@uni-rostock.de

Self-imaging photonic lattices enable perfect transfer of quantum and classical states, yet are challenging to implement at scale. We harness supersymmetry to engineer compacted two-dimensional systems exhibiting equivalent characteristics and experimentally investigate their dynamics.

Keywords: *supersymmetric optics, photonic lattices, self-imaging, state transfer, fs-laser writing*

INTRODUCTION

The evolution dynamics of wave-mechanical systems are governed by the full set of their modes and their respective eigenvalues. The key task of transferring arbitrary excitation patterns between two specific planes can therefore be accomplished by an appropriate structure of the eigenvalue spectrum. Along these lines, self-imaging is particularly effective if the spectrum is equidistantly spaced, similar to that of the harmonic oscillator. In finite-size discrete systems, the so-called J_x lattice fulfils this condition and has been employed for the perfect coherent transfer of quantum and classical states alike [1–3]. Yet, implementing large-scale J_x arrays remains experimentally challenging, as this class of systems relies on a precise realization of a large number of different yet finely tuned nearest-neighbor interaction strengths spanning a substantial dynamic range.

To overcome these limitations, we leverage the concept of supersymmetric (SUSY) photonics [4,5] and present a method to design families of compact two-dimensional equivalent systems that inherit the spectral and key dynamic features of one-dimensional J_x arrays while requiring dramatically fewer distinct coupling values [6].

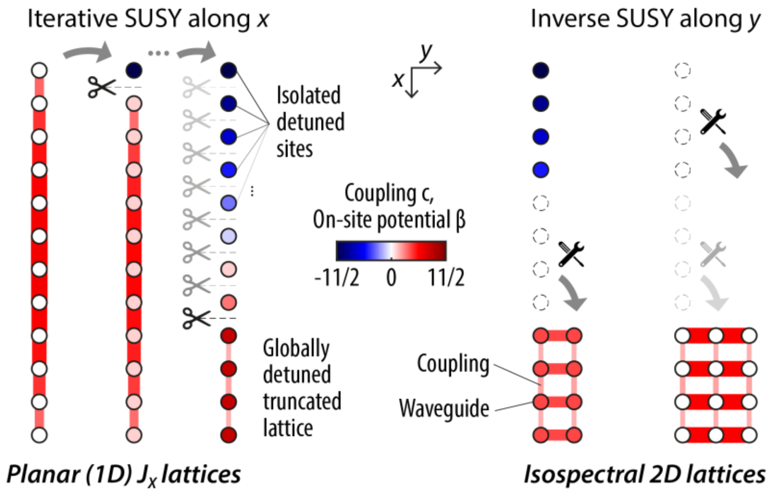

Fig. 1. Transforming a one-dimensional J_x array of twelve sites into a two-dimensional array through subsequent supersymmetry transformations. Color/linewidth indicates the relative strength of the coupling and detuning between sites.

TRANSFORMATION METHOD

Our approach relies on applying a sequence of discrete SUSY transformations to the original tridiagonal Hamiltonian of a planar J_x array to transform it into one that represents isospectral two-dimensional lattices. As example, we here consider a 12-site J_x array in Fig. 1. Applying a discrete SUSY transformation results in its first-order unbroken superpartner, a globally detuned 11-site J_x array, which retains the spectrum of the original array with the exception of a single removed eigenvalue whose eigenstate resides on an isolated site. Iteratively repeating this procedure results in a sequence of higher-order superpartners and isolated sites. Next, we reattach these sites in a series of inverse SUSY transformation steps in the orthogonal direction, thereby constructing two-dimensional arrays. The final array of 4 × 3 sites is a two-dimensional J_x array where coupling in y direction is increased compared to the x direction. This two-dimensional structure and its original one-dimensional counterpart share all their eigenvalues and will therefore exhibit similar characteristics.

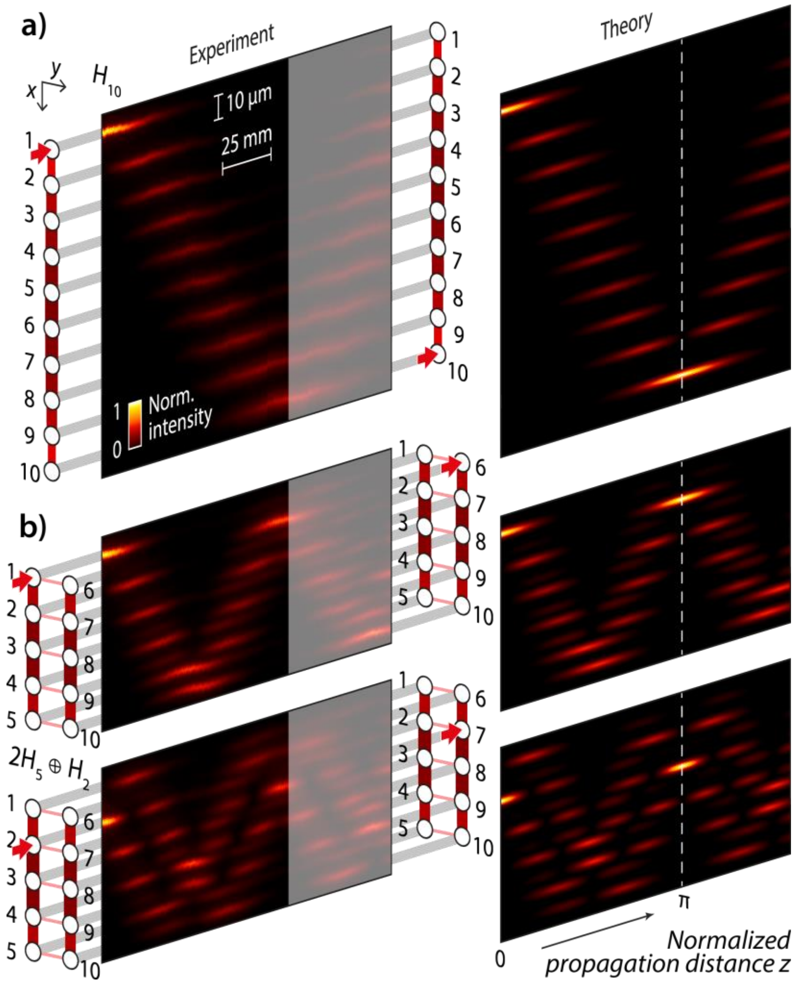

Fig. 2. Measured (left) and calculated (right) evolution dynamics of light in a) a ten-waveguide J_x array and b) its two-dimensional superpartner. In each panel, the initially excited and imaging target waveguides are marked by arrows. The imaging distance $z = \pi$ is indicated by semitransparent overlay (experiments) and dashed lines (simulations), respectively.

RESULTS

We experimentally realize these systems in arrays of evanescently coupled waveguides fabricated by femtosecond-laser direct writing [7] and investigate their imaging properties by recording the intensity dynamics of guided light through fluorescence microscopy [8]. We implement a conventional ten-waveguide J_x array and its two-dimensional superpartner of 5 × 2 sites. Figure 2 shows the observed light evolution for distinct input waveguides in each of these systems. We find that, while exhibiting systematically different dynamics in these two systems, light coalesces to the single target waveguide at the same imaging distance, demonstrating the desired perfect state transfer enabled by their identical spectra.

In conclusion, our method readily allows for the design of compact self-imaging architectures with a minimum of structural parameters. By enabling increased robustness to perturbations and fabrication inaccuracies, it provides an avenue for high-fidelity coherence-maintaining state transfer in larger-scale photonic circuits.

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