Taiji Resonators for Self-Referenced Exceptional Point Sensing

(Student paper)

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Integrated photonics can be used to create biosensors. By integrating a microring resonator with a Bragg grating, a self-referenced sensor can be created. The self-referenced sensor is a robust sensor, since it is sensitive to the presence of biomarkers, while being insensitive to temperature and bulk refractive index variations. To improve the sensitivity of the sensor, the microring resonator can be replaced by a Taiji resonator. This paper discusses the theoretical potential of this device for exceptional point sensitivity.

Keywords: *Biosensing, Self-referenced sensors, Taiji resonator, Exceptional points*

INTRODUCTION

Integrated photonics is a promising technology to create devices that scale-up non-invasive diagnostics. For example, photonic integrated circuits (PICs) can be fabricated that contain microring resonators, that are well-suited to be used as biosensors. The microring resonator (MRR) can sense changes in the environment of the device by measuring changes in the resonance spectrum. In biosensing applications, the resonator is typically covered with acceptor molecules that selectively bind to a particular biomarker. If a bodily fluid is studied, such as blood or urine, the biomarkers in the sample can attach to the acceptor molecules. As a result, the effective refractive index of the waveguide changes locally, resulting in a changed optical path length. The change in optical path length causes the resonances to shift, which can be measured.

In a standard MRR sensor, shifts in resonance frequency can be caused by a variety of factors, including changes in temperature and variations in the bulk refractive index. As a result, simply observing a shift in resonance does not necessarily mean that biomarkers have bound to the acceptor molecules. To ascribe changes in the resonance spectrum to the presence of biomarker molecules, a reference is required. The reference is used to distinguish between different signal sources. Typically two transducers, or rather MRRs, are used. One of the MRRs is sensitive to biomarker adhesion, the other is not. However, using multiple transducers can introduce uncorrelated noise. To compensate for the uncorrelated noise contributions, a self-referenced MRR can be designed.

The self-referencing behaviour of the sensor is obtained by including a waveguide Bragg grating (WBG) in the path of the cavity. The WBG acts as a reflector, lifting the degeneracy between the clockwise and counterclockwise resonator mode. As a result, the single resonances split into resonance doublets. The frequency separation of the resonances in the doublet is directly related to the WBG reflectivity. By measuring the frequency separation, instead of the absolute position of the resonances, any signal that changes the grating reflectivity can be measured [1]. By filling the trenches of the WBG with acceptor molecules, the reflectivity changes upon biomarker adhesion. Since the reflectivity is primarily affected by the biomarker adhesion, rather than the temperature and bulk refractive index variations, the MRR with WBG is said to be self-referenced. As a result, the sensor readout becomes insensitive to these temperature and bulk refractive index variations.

Self-referencing of MRRs has been demonstrated in Al_2O_3 PICs, in which the resonator cavity is equiped with a PMMA WBG [1, 2]. These devices are highly sensitive to changes in the grating reflecitivity, which is sensitive to changes in the index contrast. The index contrast changes strongly upon biomarker-acceptor binding, but it is less sensitive to temperature and bulk refractive index changes. It is believed that the sensitvity to changes in index contrast can be improved by moving to so-called exceptional point sensors [3]. In this paper, a design for an exceptional point sensor, that still exhibits self-referenced behaviour, is presented.

TAIJI RESONATORS

Exceptional points may show up in non-Hermitian systems. One of the structures that exhibits the loss-gain mechanism needed to find exceptional points is the so-called Taiji resonator. The non-Hermicity of the system can be exploited to show phenomena such as unidirectional reflection, that is an important aspect in unidirectional lasers [4]. However, the use of Taiji resonators may also pave the way for sensors with exceptional point sensitivity. At an exceptional point, the sensor read out varies with the n^{th} square root of a specific parameter, instead of linearly. The use of a Taiji resonator as a self-referenced exceptional point sensing is studied in this paper.

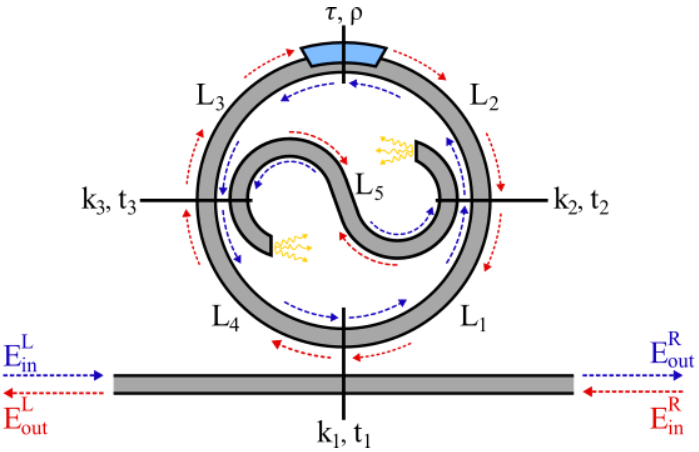

Fig. 1. Geometry of a self-referenced Taiji resonator.

To show the promising behavior of the Taiji resonator as an exceptional point sensor, as depicted in figure 1, time-dependent coupled mode theory is applied. Using coupled mode theory, the frequency splitting of a passive Taiji resonator, equipped with a Bragg grating as shown in Figure 1, can be determined by solving [3]

$$\frac{d}{dt}\begin{bmatrix} A_{cw} \\ A_{ccw} \end{bmatrix} = \begin{bmatrix} j\omega_0 - \alpha & \kappa_{12} \\ \kappa_{21} & j\omega_0 - \alpha \end{bmatrix}\begin{bmatrix} A_{cw} \\ A_{ccw} \end{bmatrix}, \tag{1}$$

In this eigenvalue problem, A_{cw} and A_{ccw} correspond to the amplitudes of the modes propagating in the clockwise and counterclockwise direction. The resonance frequency of the modes is given by ω_0, and κ_{12} and κ_{21} represent the coupling from counterclockwise (CCW) to clockwise (CW) mode and vice versa. Finally, α represents the losses experiences by the modes, including propagation losses, as well as losses due to coupling to the counterpropagating mode.

The coupling coefficient κ represents the amount of energy transferred from one mode to another per unit time. In a Taiji resonator, this coupling is not symmetric. In the S-shaped branch within the ring cavity, energy is only coupled from the CW to the CCW mode, while there is no coupling from the CCW to the CW mode through this branch. In this way, there is a loss-gain mechanism that is responsible for the exceptional point sensitivity.

The coupling per roundtrip from the CW to the CCW mode κ_{21}, and the coupling from the CCW to the CW mode κ_{12} are given by

$$\kappa_{12} = \rho\frac{v_g}{L}; \qquad \kappa_{21} = \rho\frac{v_g}{L} + \eta k_2 k_3 \frac{v_g}{L}. \tag{2}$$

In equation (2), v_g is the group velocity and L is the circumference of the outer ring structure of the Taiji resonator. The parameters ρ, k_2 and k_3 represent the grating reflection coefficient and the coupling coefficients between the outer ring and S-shaped branch, respectively. The parameter η represents the phase acquired by propagating through the S-shaped branch. Assuming that the fields in the CW and CCW mode can be described by

$$a_{cw}(t) = A_{cw}\exp(j\omega_0 t)\,; \qquad a_{ccw}(t) = A_{ccw}\exp(j\omega_0 t) \tag{3}$$

and substituting these fields in equation (1) results in

$$j\omega_0\begin{bmatrix} A_{cw} \\ A_{ccw} \end{bmatrix} = \begin{bmatrix} j\omega_0 - \alpha & \rho\frac{v_g}{L} \\ \rho\frac{v_g}{L} + \eta k_2 k_3 \frac{v_g}{L} & j\omega_0 - \alpha \end{bmatrix}\begin{bmatrix} A_{cw} \\ A_{ccw} \end{bmatrix}. \tag{4}$$

To find the resonance splitting, one can find the eigenvalues of the two-by-two matrix described by equation 4. The eigenvalues are given by

$$\omega_{\pm} = j\omega_0 - \alpha \pm \frac{v_g}{L}\sqrt{\rho^2 + \eta k_2 k_3 \rho}, \tag{5}$$

such that the resonance splitting becomes $\Delta\omega = |\omega_+ - \omega_-|$,

$$\Delta\omega = \frac{v_g}{L}\sqrt{\rho^2 + \eta k_2 k_3 \rho}. \tag{6}$$

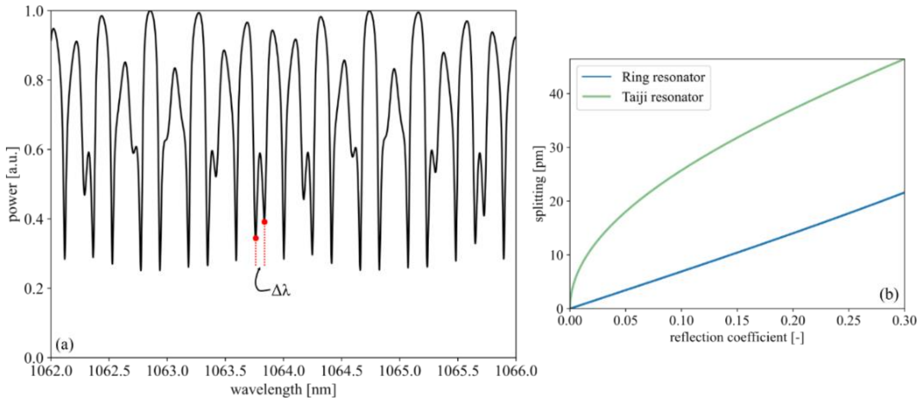

Fig. 2. Transmission spectrum of a Taiji resonator (a), evaluated for $\boldsymbol{L = 3484\ \mu m}$, $\boldsymbol{k_2 = k_3 = 0.7}$, $\boldsymbol{n_{eff} = 1.55}$, $\boldsymbol{\alpha = 0.98}$ *and* $\boldsymbol{\rho = 0.63}$. *The resonance splitting for the Taiji resonator is compared (b) with that of an MRR, where* $\boldsymbol{k_2 = k_3 = 0}$.

The frequency splitting is maximized if $\eta = 1$. In case $\rho \ll k_2 k_3$, then $\rho^2 \ll k_2 k_3 \rho$, and equation 5 reduces to

$$\Delta\omega = \frac{v_g}{L}\sqrt{k_2 k_3 \rho} \propto \sqrt{\rho}. \tag{7}$$

In this limit, the resonance splitting grows with the square root of ρ meaning that the sensor operates at an exceptional point. If the S-shaped branch would not have been there, so $k_2 = k_3 = 0$, or if the aforementioned limit does not hold, the frequency splitting would be proportional to $\Delta\omega \propto \rho$.

To support the claim that the Taiji resonator allows for exceptional point sensitivity, the full transmission function of the device is derived. The transmission function follows from applying the scattering matrix method to the waveguide structure shown in figure 1. The transmission function is given by

$$T = \left| t_1 - \frac{\left(\alpha^4 L_1 L_2 L_3 L_4 e^{j\beta L}\right)^2 \det S\, t_1 t_2^2 t_3^2 k_1^2 - \left(\alpha^5 L_1 L_2^2 L_4 L_5 e^{j\beta(L_1+2L_2+L_4)} e^{j\beta L_5}\right)\rho t_2 k_1^2 k_2 k_3 + \left(\alpha^4 L_1 L_2 L_3 L_4 e^{j\beta L}\right)\tau k_1^2 t_2 t_3}{1 + 2(\alpha^5 L_1 L_2^2 L_4 L_5 e^{j\beta(L_1+2L_2+L_4)} e^{j\beta L_5})\rho t_1 t_2 k_2 k_3 - 2(\alpha^4 L_1 L_2 L_3 L_4 e^{j\beta L})\tau t_1 t_2 t_3 - (\alpha^4 L_1 L_2 L_3 L_4 e^{j\beta L})^2 \det S\, t_1^2 t_2^2 t_3^2} \right|^2 \tag{8}$$

and a section of the spectrum is shown in figure 2a. In equation 8, α represents the propagation losses and β refers to the propagation constant of the waveguide mode. The parameter L_i, t_i and k_i correspond to the arc lengths, coupling coefficients and transmission coefficients as shown in figure 1. Finally, ρ and τ give the reflection and transmission coefficient of the WBG, while S describes the scattering matrix of the WBG. To show the improved sensitivity, the resonance splitting is monitored as the reflection coefficient changes. The resulting resonance splitting for a regular self-referenced MRR sensor and the Taiji resonator is shown in figure 2b. From this figure, it is evident that the Taiji resonator allows for a higher sensitivity with respect to changes in grating reflectivity.

CONCLUSION AND OUTLOOK

Under specific conditions, the Taiji resonator allows for exceptional point sensitivity while maintaining self-referenced behavior. Comparing the resonance splitting in a regular self-referenced MRR with the resonance splitting in the Taiji resonator, it is clear that the resonance splitting varies with the square root of the reflection coefficient. This square root dependence is an indication of exceptional point sensitivity.

The following step is to fabricate the Taiji resonators that are equipped with a WBG. After fabrication, it is important to demonstrate that resonance splitting can be observed in the Taiji resonators in practice. Afterwards, the sensitivity of the device is to be determined, such that a comparison can be made with the self-referenced MRR sensors. To continue the study of the self-referenced Taiji resonators, the devices are fabricated from Al_2O_3 and those devices are provided with a Bragg grating made from PMMA. From these first experiments, resonance splitting has been observed. Future work involves the determination of the sensitivity of the devices.

References

[1] M. de Goede et al., *Al_2O_3 microring resonators for detection of cancer biomarker in undiluted urine*, Optics Express, vol. 27, no. 13, pp. 18508, 2019
[2] B. Jongebloed et al., *Grating engineering for self-referenced Al_2O_3 microring resonator sensors*, in proceeding of the IEEE Photonics Benelux Annual Symposium, 2022
[3] Q. Zhong et al., *Sensing with exceptional surfaces in order to combine sensitivity with robustness*, Physics Review Letters, vol. 122, no. 15, pp. 153902, 2019
[4] A. Calabrese, *Unidirection reflection from an integrated 'taiji' microresonator,* Phot. Research, vol. 8, no. 8, pp. 1333, 2020.
[5] H. A. Haus, *Waves and fields in optoelectronics TT*, Prentice-Hall, Englewood Cliffs, pp. 197-234, 1984.