Nonlocal interactions in planar metal layers

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Nonlocal interactions of plasmonic nanostructures are currently being intensively investigated. It is generally accepted that nonlocal interactions are most pronounced on metallic structures with nanometre unit sizes and affect the shape of spectral functions of characterizing quantities in the resonance region. Since the numerical analysis of nonlocal phenomena is quite complicated in general. Clearly, it is advantageous to use it for the analysis of geometrically more complex structures. However, for some structures with simpler morphology, it is possible and beneficial to find (quasi)analytical solutions which can then be applied to build semi-analytical approaches for the analysis of more complex structures. Therefore, this contribution is focused on the analytical description of nonlocal manifestations of planar metal layers using the hydrodynamic model. Our aim is to present here the possibilities of constructing an efficient nonlocal model for 1D metal layers.

Keywords: Hydrodynamical model, nonlocal plasmonics, metal layer, transfer matrix

INTRODUCTION

So-called non-local phenomena in metallic structures are currently being intensively investigated. These phenomena are more pronounced on metal structures whose at least one dimension is usually nanometer-sized [1]. In very general terms, the nonlocal medium disrupts the relationship between the electric field vector and the electric induction vector in the form $\boldsymbol{D}(r,\omega) = \varepsilon(\omega)\boldsymbol{E}(r,\omega)$, which applies to dielectric materials. This deviation is caused by induced current density waves which are associated with a longitudinal electric field that does not exist in dielectric materials. In order to be able to more accurately simulate the behavior of metallic nanostructures with an incident electromagnetic wave, it is necessary to extend the classical models by an equation including the relationship between the electric field and the induced current density. The most commonly used non-local model is the so-called hydrodynamic model (HDM) which already has a number of applications and modifications [2]. For our purposes, however, a standard version of HDM in the form of equations (1) and (2) will suffice. The first HDM equation is de facto Maxwell's wave domain equation assuming that the electromagnetic wave propagates in the environment of positively charged metal ions [3] of permittivity $(\varepsilon_t - \varepsilon_{eg})$, where ε_t denotes the total permittivity of the material and ε_{eg} is the permittivity of the free electron gas. The symbol $k_0 = \omega/c$ denotes the wave number in vacuum. Equation (2) represents a linearized form of equation for the electron gas motion in the EM field in the frequency domain.

$$\nabla \times (\nabla \times \boldsymbol{E}) - k_0^2(\varepsilon_t - \varepsilon_{eg})\boldsymbol{E} = i\omega\mu\boldsymbol{J}, \qquad (1)$$

$$\beta^2\nabla(\nabla.\boldsymbol{J}) + \omega(\omega + i\gamma)\boldsymbol{J} = i\omega\omega_p^2\varepsilon_0\boldsymbol{E}. \qquad (2)$$

The values of the important constants of the HD model (Fermi velocity, plasmonic frequency, nonlocal parameter, damping coefficient) for silver and gold are given in Table 1.

constant	units	gold	silver
v_f	$10^6 m.s^{-1}$	1.39	1.39
ω_p	$10^{16} Hz$	13673	13627
β	$10^6 m.s^{-1}$	$0.774 * v_f$	$0.774 * v_f$
γ	$10^{16} Hz$	$\omega_p/127$	$\omega_p/360$

Tab. 1 Table of HD model parameters (Fermi velocity, plasmonic frequency, nonlocal parameter, damping)[3].

MODEL FOR NONLOCAL METALLIC LAYERS

One of the simplest and also the easiest structures to fabricate is a structure composed of planar metal-dielectric layers. The analysis of these structures is very valuable both for the examination of the deviations and limits of classical local theory and also for the knowledge that could help in the analysis of more complex structures in the future. Recently, some numerical simulation methods have been developed [4], but our interest here is to find a (quasi)analytical solution, specifically using the formalism of transfer matrices [5] which is a suitable tool for the analysis of planar layered structures. In addition to the advantages of simple implementation and calculation speed, analytical methods have clearly other positives, such as a relatively easy possibility of finding extremes of selected quantities or a possibility of adapting to some modifications of the HDM or boundary conditions for the current density. Our goal was to create a simple simulation tool enabling the analysis of both single metal layers and metal bilayers, surrounded by dielectrics, see Fig. 1. We focused exclusively on the description of the interaction of the TM polarized plane wave with the metallic gold or silver layer because in the case of TE polarization, the existence of nonlocal

effects does not manifest. The interaction of the TM plane wave with the nonlocal environment can be schematically illustrated in Figure 1.

As indicated in Figure 1, a TM polarized wave propagates into a metal layer at the angle of incidence α_0. The plane of incidence is determined by the X and Y axes. The first medium is a dielectric defined by the refractive index n_0 and the impedance $z_0 = \sqrt{\mu/\varepsilon}$, where μ and ε denote relative permeability and permittivity of the first medium, respectively. The parameters of surrounding media are marked analogously. The numbers in the subscript of the quantity are assigned to the surrounding medium, the lowercase letters r, s, t, i represent quantities of the reflected, scattered, transmitted, and incident fields, capitals letters T and L indicate transversal and longitudinal waves. The additional superscripts (+) and (–) of the wave number are tied to the direction of propagation along the Z axis. Tangent fields in the y-axis direction at the first interface are denoted as $\boldsymbol{E_{01}}$ and $\boldsymbol{H_{01}}$, similarly tangent fields at the second interface are denoted as $\boldsymbol{E_{12}}$ and $\boldsymbol{H_{12}}$.

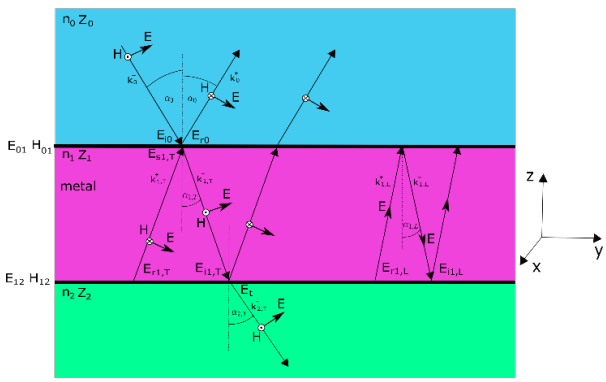

Fig. 1 Scheme of nonlocal interaction of metal layer with TM-polarized plane wave.

All the characteristic properties of a considered metal layer can be determined by solving the boundary conditions of the continuity of tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ at both boundaries together with the conditions $\boldsymbol{J_n} = \boldsymbol{0}$, i.e. the zero normal components of the current density. From equations (1) and (2), it is possible to determine the wave numbers for the transverse and longitudinal fields and a specific form of the boundary conditions can be determined from the geometric concepts, see Fig. 1. In further relatively laborious steps, it is possible to obtain relationships between the magnitudes of the tangential fields at both interfaces of the metal layer and thus obtain a transfer matrix, expressed with a general expression (3) (the symbol η_0 means the vacuum impedance):

$$\begin{pmatrix} \boldsymbol{E_{01}} \\ \eta_0 \boldsymbol{H_{01}} \end{pmatrix} = M_1 \begin{pmatrix} \boldsymbol{E_{12}} \\ \eta_0 \boldsymbol{H_{12}} \end{pmatrix}. \qquad (3)$$

The transfer matrix of the resulting multilayer structure, i.e. the structure with isolated metal layers, can be expressed using the transfer matrices of the individual layers as

$$M_{tot} = M_1 * M_2 \dots \dots . M_n. \qquad (4)$$

The index numbering of the layers and their transfer matrices is considered ascending in the direction of propagation of the incident wave. The situation is much more complicated in the case of two bounded metal layers. The boundary condition for the normal components of the current density at the joint interface of the two metals no longer applies and even relation (4) for the total transfer matrix cannot be used. However, in general, if the total transfer matrix of a given multilayer is already known, then the individual characteristic quantities of the whole structure can be determined as an analytical function of the components of the total transfer matrix.

RESULTS

We have built an analytical method based on the above-mentioned principles, which can analyze the non-local effects of both metal layers and metal bilayers. Our research has so far been limited to finding the parameters of the structure and the incident TM plane wave in which the nonlocal response is more pronounced. Fig. 2 shows two cases of analyzed structures: (a) a single metal layer and (b) a metal bilayer which we investigated.

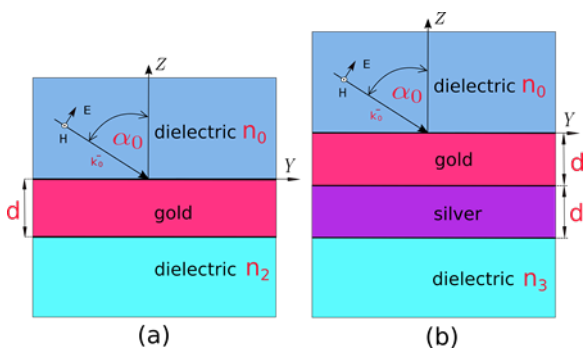

Fig. 2 Illustration of structure and main parameters of (a) single metal layer and (b) double metal layer.

In order to assess the effect of the nonlocal response, we chose the deviation between reflectances $(R_{HD} - R)$, where R_{HD} is the reflectance calculated using the HD nonlocal model while R is the reflectance determined in the standard (local) way. Below we present selected calculations of the reflectance deviation for the gold metal layer (see Figs. 3 and 5) and their analogues (see Figs. 4 and 6) for the metal bilayer (for simplicity selected here as: the upper layer as gold, lower layer as silver, same thickness). From our extensive simulations, it turns out that a more significant deviation due to nonlocal behavior can

be expected mainly for the sliding angles of incidence (the angle 85.5 degrees chosen as the optimal value).

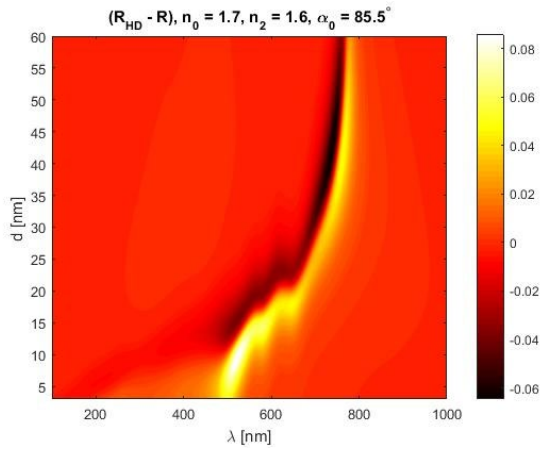

Fig. 3. Reflectance deviation between HD and local model as dependence on gold layer thickness $\boldsymbol{d}$ *and wavelength* $\boldsymbol{\lambda}$.

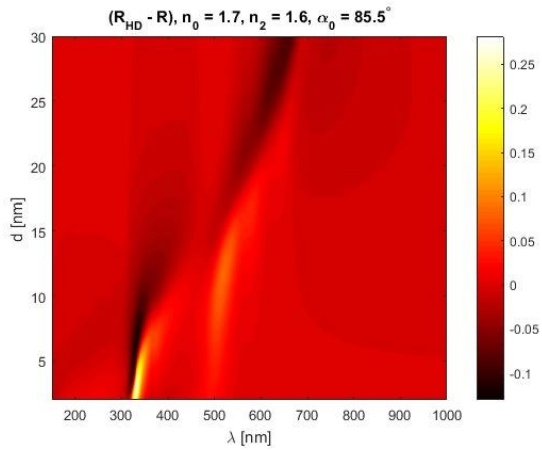

Fig. 4. Reflectance deviation between HD and local model as dependence on total gold-silver layer thickness $\boldsymbol{d_{tot}}$ *and wavelength* $\boldsymbol{\lambda}$.

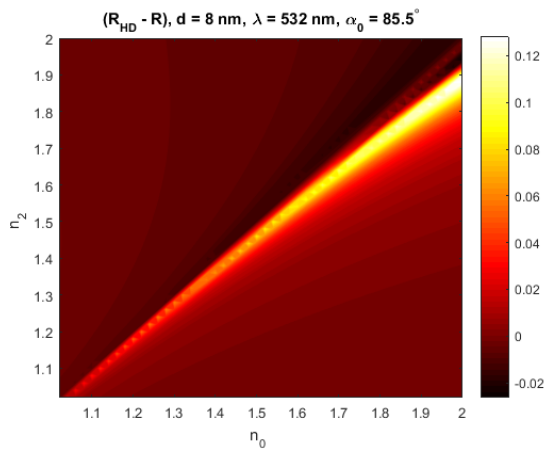

Fig. 5. Reflectance deviation between HD and local model for gold layer at wavelengt $\boldsymbol{\lambda = 532\ nm}$ *as dependence on upper* $\boldsymbol{n_0}$ *and bottom* $\boldsymbol{n_2}$ *refractive indices.*

Figs. 3 and 4 show the dependence of the reflectance deviation on the wavelength of the incident wave and the thickness d of the gold layer (d_{tot} for the gold-silver bilayer in Fig. 4), for the refractive indices of the media of $n_0 = 1.7$ and $n_2 = 1.6$ (or n_3, respectively).

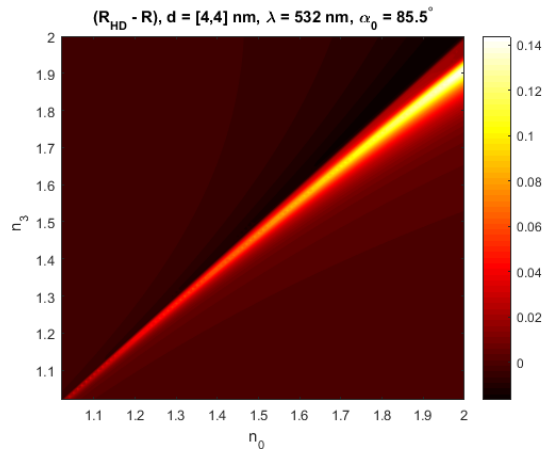

Fig. 6. Reflectance deviation between HD and classic model for gold-silver bilayer at wavelengt $\boldsymbol{\lambda = 532\ nm}$ *as dependence on upper* $\boldsymbol{n_0}$ *and bottom refractive index* $\boldsymbol{n_3}$.

The deviation of the reflectance is fundamentally influenced by the refractive indices of the external environment, such dependence at a fixed thickness of the gold layer and the total thickness of the bilayer at 8 nm (@ wavelength of 532 nm) is shown in Figs. 5 and 6, respectively. From the analysis presented here, it is evident that in specific situations, for certain parameters, non-local effects can manifest themselves in a significant way for thin metal layers and bilayers.

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