Design of autocorrective interferometers using the Bloch sphere

Matteo Cherchi

VTT – Technical Research Centre of Finland
matteo.cherchi@vtt.fi

The Bloch sphere is a very powerful tool to design two-path interferometric systems, providing superior physical insight into the working principle of autocorrective devices like broadband 50:50 splitters or flat-top interleavers, also enabling the derivation of simple analytical design formulas. I will first introduce a few examples of design applications to then present experimental realisations of the same on our silicon photonics platform.

Keywords: *integrated interferometers, Bloch sphere, autocorrective photonic devices, flat top filters, wavelength insensitive splitters, photonic integrated circuits*

GEOMETRIC REPRESENTATION OF TWO-PATH INTEFRFEROMETRIC SYSTEMS

Two-path interferometric devices are ubiquitous in photonic integrated circuits. They are simply built out of just two basic building blocks: power splitters – including directional couplers and multimode interferometer (MMI) splitters – and phase shifters – including imbalanced waveguide lengths or waveguide tapering [1]. These systems can be treated analytically based on a transfer matrix formalism, but significant physical insight can be gained when representing them on the so-called Bloch sphere [1]. All possible combinations of power splitting ratio and relative phase can be mapped on the sphere as depicted in Fig. 1. Therefore, any state is unequivocally determined by two angles: ϑ representing the phase shift and 2α corresponding to the splitting ratio $\sin^2(\alpha)/\cos^2(\alpha)$. The three Stokes parameters S_1, S_2, and S_3 correspond to the three orthogonal axes. The intersections of S_1 with the sphere represent the single uncoupled waveguide modes E_1 and E_2, the intersections of S_2 represent the eigenmodes of synchronous directional couplers, i.e., the symmetric and anti-symmetric modes E_S and E_A, whereas the intersections of S_3 represent the superpositions in quadrature and anti-quadrature E_R and E_L. Noticeably, the sphere is not only a tool to show the states as static points, but also a powerful way to show their evolution under the action of power splitters and phase shifters, which operate as simple rotations on the sphere. A synchronous directional coupler acts as a rotation around S_2, which is the axis of its eigenmodes, i.e., the symmetric and anti-symmetric modes E_S and E_A. Similarly, a phase shifter acts as a rotation around S_1, which is the axis of its eigenmodes, i.e., the single waveguide modes E_1 and E_2. More in general, an asynchronous coupler acts as a rotation about the axis of its eigenstates [1], i.e., an axis laying in the equatorial plane somewhere between S_1 and S_2.

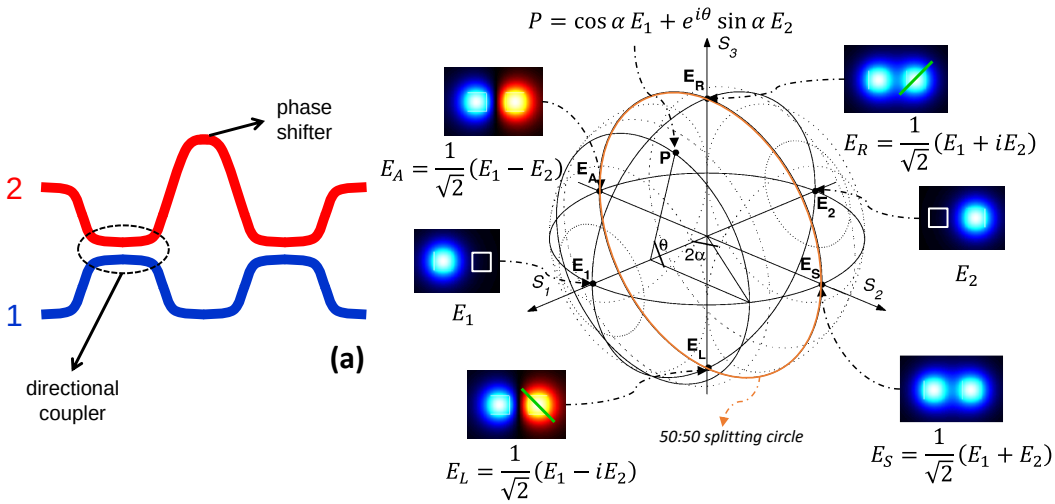

Fig. 1 a) Schematic layout of a simple 2-path interferometer; b) the Bloch sphere as a geometric representation of all possible states of a two-path system, showing the eigenmodes corresponding to the axes S_1, S_2 and S_3 and the two angles α and ϑ associated with a generic point P on the sphere. The angle α indicates the power fractions $\cos^2\alpha$ and $\sin^2\alpha$ in the single waveguide modes E_1 and E_2 respectively, whereas ϑ shows the relative phase between the two paths.

FLAT-TOP 50:50 SPLITTERS

Little and Murphy [2] have reported a maximally flat 50:50 splitter in the form of a generalised Mach-Zehnder interferometer (MZI) composed by a 50:50 ("half") coupler and a 100:0 ("full") coupler (the first number representing the percentage in the cross port) with a 120° phase shifter in between them (see Fig. 2), but their

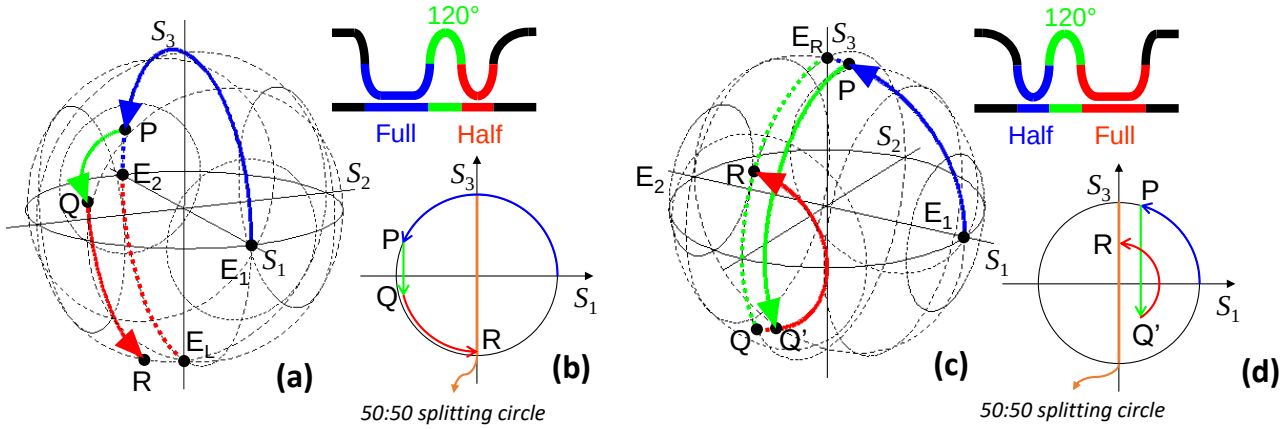

Fig. 2. a) FH configuration and its trajectory on the Bloch sphere. The dashed lines correspond to the nominal case, whereas continuous lines show the autocorrective action when the coupling angles are reduced by 10%; b) projection of the same trajectory on the S_1S_3 plane c) and d) same as a) and b) but for the HF configuration.

paper did not disclose how the result had been derived. The working principle of the flattened MZI 50:50 splitter can be easily understood using the geometrical representation. Let's first consider the configuration where the full coupler comes first, which we will call the FH configuration (Fig. 2a). At the nominal wavelength, the full coupler will simply switch all the power to the cross port, i.e., rotate the state by 180° from E_1 till the antipodal E_2. In this particular case, the phase shifter rotation corresponds to no changes (E_2 lies on S_1 or, in other words, it is an eigenstate of the phase shifter), and the last 50:50 directional coupler simply rotates the state by 90° to reach the south pole state E_L, i.e., perfect 50:50 splitting (see dashed trajectory in Fig. 2a). Things become more interesting off the nominal wavelength, for example at the shorter wavelength corresponding to a reduction of 10% in the coupling angle. In this case, the full coupler will reach only till the point P, just above E_2 (blue continuous line in Fig. 2a). This time the 120° phase shifter brings P to Q that, by construction, has a distance from the equatorial plane that is half the distance of P, but with opposite sign. This compensates exactly for the 10% reduction in the last coupling rotation, given that the reduction of a half coupler must be half that of a full coupler. Therefore, the ending point R belongs to the circle in the S_2S_3 plane, which is the locus of all the 50:50 states. This ensure a flat 50:50 amplitude response. Nevertheless, R departs from the south pole E_L by an angle proportional to the departure of the coupling angle from its nominal value, resulting in a linear variation of the relative phase. In a similar fashion, we can appreciate the working principle of a HF configuration, where the half coupler comes first (Fig. 2c).

In this case the nominal trajectory reaches the north pole E_R first, then point Q in the southern hemisphere (with -150° phase shift) and finally point R (with 150° phase shift). All points E_R, R and Q belong to the 50:50 circle. At a shorter wavelength corresponding to, e.g., 10% reduction of the coupling angle, the trajectory changes, but the end points R remains substantially the same. This is because the 120° phase shift ensures that the red rotation occurs on a circle with half the radius of the blue rotation. As a result, the blue arc and the red arc are shortened exactly by the same length because the coupling angle changes proportionally to the angle itself and, by construction, the coupling angle of the full coupler is twice the angle of the half coupler. Therefore, this configuration ensures flat response for amplitude and phase at the same time. This difference between the HF and FH configuration becomes very important when designing integrated Michelson interferometers [3].

FLAT-TOP FILTERS

The geometrical representation can also easily explain the working principle of two-stage generalised lattice filters that are cascades of couplers and phase shifters where the imbalance between the two arms doubles at each stage. The usual design approach is based on numerical optimization algorithms to determine the coupling coefficients of the splitters. Let's consider one of the examples reported by Madsen and Zhao [4], with couplers of 0.500, 0.7143, and 0.9226, and phase shifters of order +1 and +2. We sketch in (a) a possible physical realisation of the filter. The transfer matrix response of the filter is plotted in Fig. 3(e). Central wavelengths of transmission bands in the cross-port correspond to a $\pi+2k\pi$ (k integer) phase shift in the first phase shifter, whereas for the bar port, central wavelengths correspond to $2m\pi$ (m integer) phase shift in the first phase shifter. For wavelengths slightly off the central wavelengths, the phase shift will depart from multiples of π, which, in the case of simple MZIs, leads to reduced transmission and extinction ratio, i.e., to non-flat response. Instead, the 2-stage filter is designed such that the phase offset in the first stage is compensated by the offset in the second stage, resulting in autocorrective behaviour and consequent flat response. In fact, the angular offset in the second stage is, by construction twice as the angular offset in the first stage, but it occurs on a circle that has about half the radius, and in the opposite

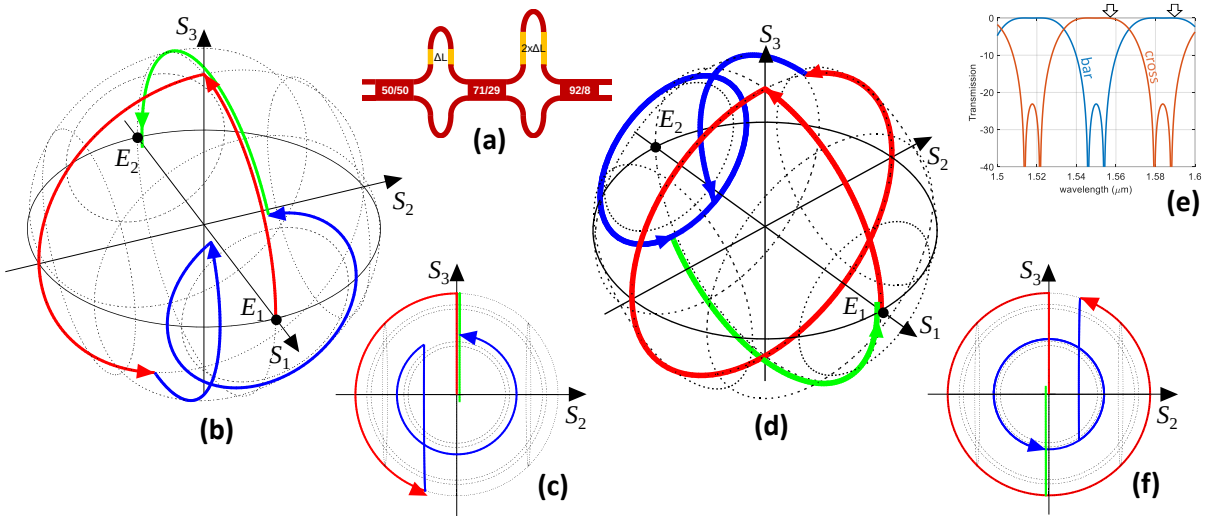

Fig. 3. a) two-stage filter under study, b) and d) show the autocorrective trajectories ensuring the flat-top response simulated in (e). c) and f) show the projections on the S_2S_3 plane, highlight the autocorrective behaviour.

direction. This is shown in Fig. 3 for the cross port and bar port respectively. The half radius is guaranteed by having chosen a 71:29 splitter, i.e., very close to 75:25 that would correspond to exactly 120° rotation. This way, the physical offsets cancel out, having same magnitude and opposite sign. More in general, thanks to the geometric representation one can derive analytic expression to identify all possible working configurations. If, like in the previous example, we want $\pi+2k\pi$ phase shifts and $2m\pi$ phase shifts to correspond to the cross state and to the bar state respectively, the rotation angles on the sphere ϕ_1, ϕ_2, and ϕ_3 corresponding to the three splitters must obey the simple system of linear equations

$$\begin{cases} \phi_1 + \phi_2 + \phi_3 = 2m\pi \\ \phi_2 - \phi_1 + \phi_3 = (2k+1)\pi \\ |\sin\phi_1| = 2|\sin(\phi_2 - \phi_1)| \end{cases} \quad \text{with solution} \quad \begin{cases} \phi_1 = \frac{\pi}{2} + p\pi \\ \phi_2 = \frac{-3\pm1}{6}\pi + q\pi \\ \phi_3 = \mp\frac{\pi}{6} + t\pi \end{cases},$$

where p, q and t are integers. In particular, the solution $\phi_1 = \pi/2$, $\phi_2 = 2\pi/3$, and $\phi_3 = 5\pi/6$ corresponds to the filter in Fig. 3. Some solutions require phase shifters with same sign and other ones with opposite sign. Another set of solutions can be also derived assuming that the cross (bar) state correspond to even (odd) phase shifts.

Eventually, in Fig. 4 we show the measured flat-top spectral response of similar devices designed using the Bloch sphere based on single multimode interferometers [5], and fabricated on VTT thick SOI platform.

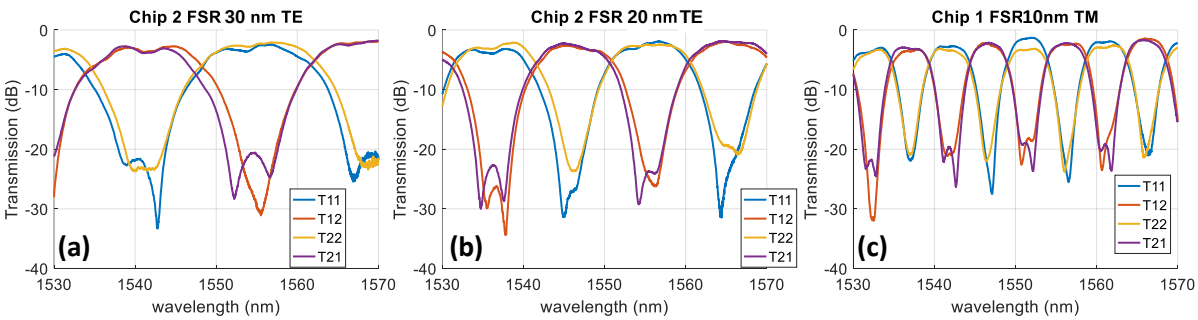

Fig. 4. Measured spectral response of three different two-stage interleavers with different FSR.

Acknowledgements: This work has been supported by the Academy of Finland PREIN, decision number 320168.

References

1. M. Cherchi, "Wavelength-Flattened Directional Couplers: A Geometrical Approach," Appl. Opt. **42**, 7141 (2003).
2. B. E. Little and T. Murphy, "Design rules for maximally flat wavelength-insensitive optical power dividers using Mach-Zehnder structures," IEEE Photonics Technol. Lett. **9**, 1607–1609 (1997).
3. M. Tormen and M. Cherchi, "Wavelength-flattened directional couplers for mirror-symmetric interferometers," J. Light. Technol. **23**, 4387–4392 (2005).
4. C. K. Madsen and J. H. Zhao, *Optical Filter Design and Analysis: A Signal Processing Approach* (Wiley, 1999).
5. M. Cherchi, F. Sun, M. Kapulainen, M. Harjanne, and T. Aalto, "Flat-top interleavers based on single MMIs," in *Silicon Photonics XV* (International Society for Optics and Photonics, 2020), Vol. 11285, p. 112850G.