Topological control of light spectrum using dynamically modulated optical waveguides

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The control of the frequency spectrum of light is a ubiquitous problem in physics and engineering. Fast telecommunication protocols rely on frequency multiplexing, and the indistinguishability of photons, strictly related to their spectral properties, is a necessary requirement for the entanglement in photonic quantum technologies. Most of the available protocols for the modification of the spectral properties of a classic light signal or single photon wave-packets are based on non-linear wave mixing, which is intrinsically stochastic and strongly dependent on the nonlinear susceptibility of the employed material. Most recently, dynamical modulation has emerged as an alternative platform to manipulate the frequency of light trapped in resonating structures [1] or travelling in isolated waveguides [2]. In this work, we theoretically inspect the dynamical modulation of homogeneous 1D arrays of optical waveguides using a refractive index perturbation caused by a sound wave travelling in a material with strong acousto-optic response. With respect to previous works, the model presented here is based on cavity-less design that allows to work with arbitrarily broadband signals. Moreover, the study of 1D chains enables the observation of topological phenomena unavailable in isolated waveguides such as disorder protected unidirectional conversion of light frequency.

DYNAMICAL MODULATION OF AN OPTICAL WAVEGUIDE

We consider paraxial light propagation in a weakly guiding optical waveguide such as the ones fabricated on fused silica substrate using femtosecond direct laser writing technique [3]. Considering a reasonably small frequency range, the waveguide supports single mode propagation over a continuous frequency spectrum ω with dispersion relation $\kappa_l(\omega)$. We now include, as depicted in fig. 1a, a monochromatic travelling-wave refractive index perturbation $\Delta n(z,t) = 2b\cos(\boldsymbol{k}_m \cdot \boldsymbol{r} - \Omega_m t)$, that can be generated by a sound wave propagating in a material with strong acousto-optic response. To gain control with respect to the longitudinal wavevector of the perturbation, we consider the sound propagating with an angle θ with respect to the optical waveguide axis so that the longitudinal wavevector can be written as $k_m^z = k_m \cos(\theta)$. This condition could be put in practice by enforcing a specific phase relation between individual sound emitters distributed in an array. Employing a linear expansion of $\kappa_l(\omega)$ around a carrier frequency ω_0 (figure 1b), the equation describing the evolution of the spectral components of an impinging light wave packet can be expressed [4] in the form of a Schrodinger-like equation in a continuous 1D frequency space with long-range interactions between frequency components separated by the dynamical modulation frequency Ω_m:

$$i\partial_z E(z,\omega) = V_s \omega E(z,\omega) - \beta E(z,\omega \mp \Omega_m)\ . \tag{1}$$

In equation (1), the hopping amplitude $\beta = k_0 b$ is proportional to the refractive index perturbation and the diagonal term $V_s = \Delta k/\Omega_m$ is proportional to the phase mismatch $\Delta k = k_m^z - \Omega_m \partial\kappa_l/\partial\omega$ between the z-component of the refractive index perturbation and the light mode and represents a (static) scalar potential along the frequency dimension. Assuming a monochromatic light input at frequency ω_0 the symmetry of the problem allows to restrict the study to a discrete set of frequencies $\{\omega_n\} = \omega_0 + n\Omega_m$ and equation (1) can be fully discretized into the following

$$i\partial_z a_n(z) = \Delta k n a_n(z) - \beta a_{n\mp 1}(z)\ . \tag{2}$$

In the case of phase matching $\Delta k = 0$ both eqn. (1) and (2) represent a system in which the dynamical modulation induces a coupling between different frequencies which ultimately modifies the spectral property of a signal travelling in the modulated waveguide. If we consider a narrowband input spectrum the dynamics in the synthetic space is well described by eq. (2) in which only one component $a_n(z) \neq 0$ for $z = 0$. As illustrated in figure 1c the diffraction pattern obtained is typical of a discrete system. On the contrary, if the spectrum of the input signal is much broader than the dynamical modulation frequency Ω_m, equation (1) describes a continuous 1D space with short range interactions and the diffraction pattern is typical of propagation in a continuous space.

In this regime the time duration of the input signal is much shorter than the period of the dynamical modulation and, as we discuss in [4], the arrival time of the light pulse with respect to the dynamical modulation can be used to control the dynamics in the frequency dimension. Indeed when the spectrum of the input light pulsw encompasses several sites of the synthetic space lattice, the phase relation between these frequency components (the so-called frequency momentum) plays the same role of the momentum for a quantum particle, or the transverse wavevector for light in waveguide arrays. As the reciprocal space of frequency is time, frequency momenta can be realistically excited with a simple time delay of the light pulse.

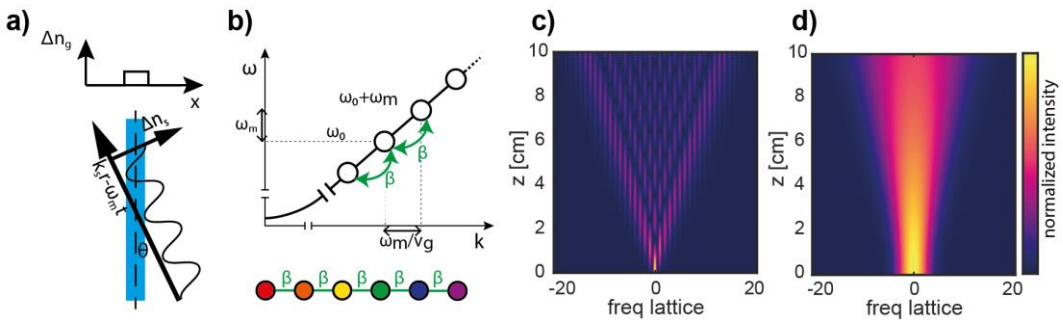

Fig. 1.(a) schematic of an optical waveguide with a travelling wave refractive index modulation. (b) spectral lattice induced in the continuous dispersion relation of the waveguide fundamental mode. (c,d) Spectral power density as a function of the propagation distance for (c) a narrowband ($\Delta\omega = 0.3\omega_m$) and (d) a broadband ($\Delta\omega = 3\omega_m$) light input in a dynamically modulated waveguide

TOPOLOGICAL EFFECTS IN A DYNAMICALLY MODULATED 1D HOMOGENEOUS CHAIN

To fully exploit the potential of our model in terms of topological robustness it is necessary to increase the dimensionality of the system. To this aim we consider an array of identical, equally spaced, optical waveguides placed at positions $x_\ell = \ell d$ subject to the same dynamical modulation propagating with an angle θ with respect to the z-axis $\Delta n(x,z,t) = 2b\cos(k_m^z z + k_m^x x - \Omega_m t)$, where $k_m^z = k_m\cos(\theta)$ and $k_m^x = k_m\sin(\theta)$ are, respectively, the longitudinal and transverse components of the modulation wavevector. Under tight binding approximation for the isolated waveguide modes, the equation describing the evolution of the spectral components of light travelling in the ℓ-th waveguide of the array can be written as:

$$i\partial_z E_\ell(z,\omega) = V_s\omega E_\ell(z,\omega) - \beta e^{\pm i\Phi_\ell} E_\ell(z,\omega \mp \Omega_m) - \xi E_{\ell\pm1}(z,\omega), \quad (3)$$

Where ξ represents the coupling coefficient between closely placed waveguides and $\Phi = k_m\sin(\theta)d$ is proportional to the transverse component of the modulation wavevector. As already mentioned in the previous section, a full discretization of the frequency space leads to a lattice version of equation (3)

$$i\partial_z a_{\ell,n}(z) = n\Delta k a_{\ell,n}(z) - \beta e^{\pm i\Phi_\ell} a_{\ell,n\mp1}(z) - \xi a_{\ell\pm1,n}(z), \quad (4)$$

Equation (4) describes a rectangular lattice in a synthetic 2D space composed by a spatial dimension (the waveguide index) and a synthetic dimension associated with the light frequency. As illustrated in figure 2a, the nonreciprocal phase factor in the coupling term along the frequency dimension provides a linear gauge field that, under Peierls substitution, is analogous to a magnetic field perpendicular to the 2D plane. As a result, eq. (4) is analogous to a Harper Hofstadter model characterized by a flux-per-plaquette of Φ.

A well-known property of the Harper Hofstadter lattice is than, whereas the flux takes rational values $\Phi = p/q$, with p and 1 co-prime integers, the spectrum is divided into exactly q well separated bands with topologically nontrivial properties. As a result, topological states appear within each bandgap that can be used to transport light in a unidirectional fashion. As an example, we consider a $\Phi = 1/4$ Harper Hofstadter model. In figure 2b we plot the dispersion relation of such lattice by considering a truncation along the real space and an infinite extension along the frequency space. The states in fig 2b are color coded accordingly to their localization in the spatial dimension, as highlighted in figure 2a. At each bandgap only one couple of states with well-defined and opposite group velocities exist, with exponential localization at opposite spatial terminations of the waveguide array. Counterpropagating states cannot scatter into one another because of opposite edge localization therefore elastic backscattering is inhibited and, once excited, these states can be used to shift the light frequency in a unidirectional and topologically protected way, as depicted in figure 2c. Crucially, for each value of the *frequency momentum* k_ω only one topological mode exists at each spatial termination. Therefore, it is possible to excite a desired red(blue)-shifting topological mode by simply injecting a light pulse in the proper edge waveguide with the right time delay. Of particular interest for applications, we note that each edge waveguide supports both blue and red-shifting topological modes with a relative difference in the frequency momentum of $\Delta k_\omega = \pi/\Omega_m$. If a train of pulses with a time difference equal to Δk_ω is injected into an edge waveguide, the even and odd pulses will excite, respectively blue- and red-shifting topological modes leading to spectrally separated pulses at the output of the device. This mechanism can have several applications in telecommunication technology for the realization of time-frequency (de)multiplexing devices or in quantum technologies for manipulating the entanglement between the time, frequency and spatial variables of photons

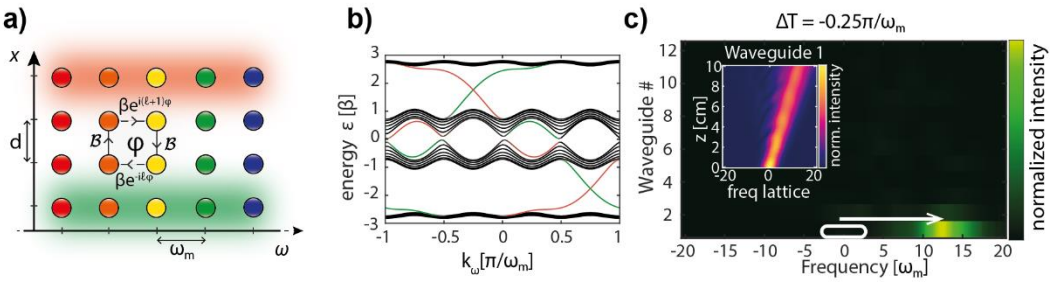

Fig. 2. a) Schematic representation of the synthetic 2D position-frequency space emerging from a dynamically modulated 1D Waveguide array. b) dispersion relation of a ¼ Harper Hofstadter lattice. Topological edge states are color-coded accordingly to their localization in the synthetic 2D space as in (a). c) Robust frequency up-conversion mediated by a topological state in the synthetic space

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