

# Robust Architecture for Programmable Universal Unitaries

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## ABSTRACT

The key element of the linear optical quantum computer is an optical circuit realizing the sequence of transformations defined by the required quantum algorithm. The most prominent approach of realizing such circuits is the reconfigurable integrated photonics. Here we study a novel universal linear-optical circuit design comprised of the series of multimode mixing blocks followed by the phaseshifting elements. We provide strong evidence that the scheme can be used as a universal interferometer even when the block's transfer matrices are chosen at random, making it virtually insensitive to errors. The proposed unitary composer can be straightforwardly implemented using standard integrated photonic technologies. We provide guidelines for appropriate multiport beamsplitter fabrication using integrated waveguide lattices.

**Keywords:** Universal reconfigurable unitaries, linear optical processing.

## 1 INTRODUCTION

Linear transformations of multiple channels take place in many applied fields, as well as in fundamental research. In integrated photonics linear multichannel transformations mathematically describe the action of an optical multichannel interferometer on a photonic input state. Universal multiport interferometers – devices capable of performing arbitrary unitary transformations – are of special interest as recent works have demonstrated their versatility in a variety of applications including microwave photonics [1], optical networking [2] and signal processing [3], photonic neural networks [4].

Furthermore, universal optical interferometers are indispensable components in linear optical quantum computation, which is a promising physical platform employing single photons and linear-optical circuits to perform quantum computing algorithms [5]. As it was previously shown, linear-optical quantum systems are able to perform a wide range of quantum computing tasks from well-known algorithms [6] to more specific ones, such as boson sampling [7] and variational algorithms [8]. The efficient realization of the latter two classes of algorithms was made possible by the universal programmable linear-optical interferometers [9].

The universal programmable interferometer structure implies the ability to set arbitrary unitary transformation of the input optical modes by tuning the phaseshifting elements inside the interferometer. Nowadays the most widely used architectures of a universal interferometer are based on the unitary matrix factorization theorem [10] which was adopted to the optical scenario [11],[12]. The essential building block in these architectures is a tunable beamsplitter which is characterized by its reflectivity and the relative phase-shift between the modes at the output. Universal optical interferometers based on [10] include a sequence of tunable beamsplitters, usually realized as Mach-Zehnder interferometers, coupling the modes according to the chosen geometry: triangular [11] or square [12]. However, both architectures strongly rely on the manufacturing quality of its comprising components - beamsplitters - to be strictly 50 : 50 balanced. Even slight deviation in the splitting ratio leads to noticeable decrease in the overall fidelity of the transformation [13]. This circumstance inevitably limits the future practical applications of these architectures for realizing high-dimensional transformations.

Here we investigate an alternative structure of universal interferometer, which is considerably different from the previously reported schemes based on meshes of two-channel blocks. Our circuit design is consisting of fixed multichannel mixing blocks and variable phaseshifting elements. Surprisingly, our design turns out to be extremely robust to perturbations in the transfer matrices of the multichannel blocks, as we show below, they may be chosen almost at random, and the desired transformation can be recovered on the manufactured device by tuning the phase shifts only. Furthermore, the placement of the variable phase shifts in the scheme can also be chosen arbitrary. Since no special assumptions about the scheme topology is implied in the analysis, a wide variety of technologies can be used to realize it in practice, for example, those based on frequency and temporal encoding, that may potentially enlarge the scale of the universal interferometric schemes [14].

## 2 SCHEME

The overall structure of the proposed unitary  $U(N)$  composer in general case, as it is depicted in Fig. 1, may be written as:

$$U = P_{K+1}V^{(K)}P_KV^{(K-1)}\dots P_2V^{(1)}P_1 \quad (1)$$

where  $V^{(m)}$  is a static mode mixing element, which in general can vary for different layers  $m$ , and  $P_m$  are variable layers consisting of independent single phase shifts:  $P_m = \text{diag}(e^{i\phi_{m1}}, e^{i\phi_{m2}}, \dots, e^{i\phi_{mN-1}}, 1)$ . The

decomposition (1) enables a variety of schemes, each having different number of layers  $K$  at a given  $N$ . Here, we describe the case of  $K = N$ . Total number of the independent variable elements equals to  $N^2 - 1$ , which is necessary to fully parametrize the  $U(N)$  space.

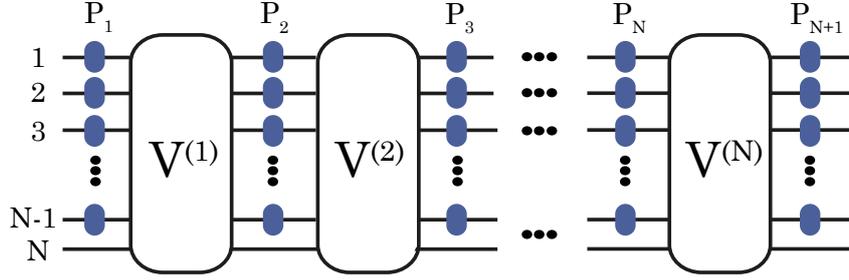


Figure 1. Proposed structure (1) layout for case  $K = N$ :  $N$  steps of mode mixing layers  $V^{(m)}$  each surrounded by  $N + 1$  phaseshifting layers. Controlling the  $N^2 - 1$  phaseshifts actively makes the interferometer reconfigurable in the whole unitary space  $U(N)$ .

### 3 RESULTS AND DISCUSSION

In order to show that the proposed design of the  $N$ -port interferometer can realize an arbitrary unitary transformation  $U(N)$  we conducted a series of numerical experiments providing a strong evidence of the universality of the current architecture.

The procedure of the numerical tests was the following: large number of random unitary sample matrices of particular dimension  $N$  was sampled from a Haar uniform distribution -  $U_s(N)$  - and for all of them optimization algorithm (we used the basinhopping algorithm from SciPy Python library) was launched to set the phases of the proposed device (1) in order to minimize the infidelity function  $1 - F(U_s, U)$  between sample matrix  $U_s$  and our arbitrary composer  $U$ :

$$F(U_s, U) = \frac{1}{N^2} |Tr(U^\dagger U_s)|^2, \quad (2)$$

The values of chosen fidelity function  $F(U_s, U)$  vary from zero to one, denoting that matrices  $U_s$  and  $U$  are completely orthogonal or exactly the same up to a constant phase.

Complete results of the numerical experiments are presented in detail in the original letter [15] and show solid evidence that the proposed structure is clearly universal. One of the results from [15] of optimization procedure for the case of a fixed random transfer matrices of the mixing layers is presented in Fig. 2, which distinctly demonstrates that in (1) almost any arbitrary unitary matrix  $V^{(m)}$  may be used as a mode mixing layer without losing the ability of the whole scheme to realize any unitary transformation, which is a key feature of the proposed multipoint interferometer design. This result demonstrates superior robustness to the fabrication imperfections of the comprising elements inside the proposed unitary composer as opposed to the previously reported implementations [13].

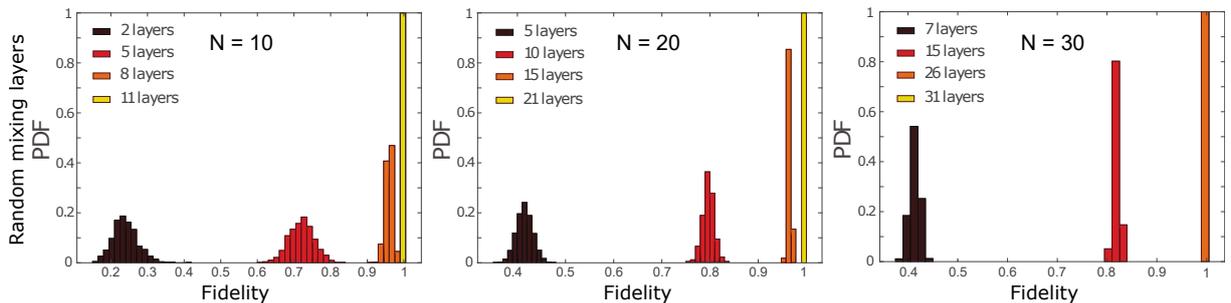


Figure 2. Results of numerical experiment. Each histogram represents the probability density function of optimizing a chosen random  $N \times N$  unitary using a composer (1) with a specified number of phase layers. The result for the full configuration clearly indicates that the proposed circuit (Fig. 1) may approximate any arbitrary unitary [15].

Integrated photonic technologies provide all the necessary instruments to develop multipoint beamsplitters which can be employed in the proposed unitary composer circuit (1). For instance, one of the simplest possible mode mixing layer configuration that can be easily exploited in linear optics is a linear array of adjacent waveguides, which is schematically illustrated in Fig. 3 (a).

To check whether this mode mixing layer configuration may be used in the proposed unitary composer we performed analogous numerical experiments discussed earlier with different interaction area lengths  $L$ , i.e. for unitary composer (1) with mode mixing layers  $V^{(m)}$  substituted with matrices of a linear array of

adjacent waveguides with a particular interaction length  $L$ . The simulation results are presented in Fig. 3 (b). The numerical simulation testifies that from certain interaction length  $L^* \approx 0.62$  mm average fidelity of the composer's operation reaches its maximum possible value  $F_{mean} = 1$  with the computational accuracy  $10^{-6}$ . Calculations were performed for  $N = 5$  equally coupled waveguides with coupling constant  $C = 0.5 \text{ mm}^{-1}$  (which is a value achievable in the experiment), each point is a mean fidelity over 40 sample random unitary matrices approximation.

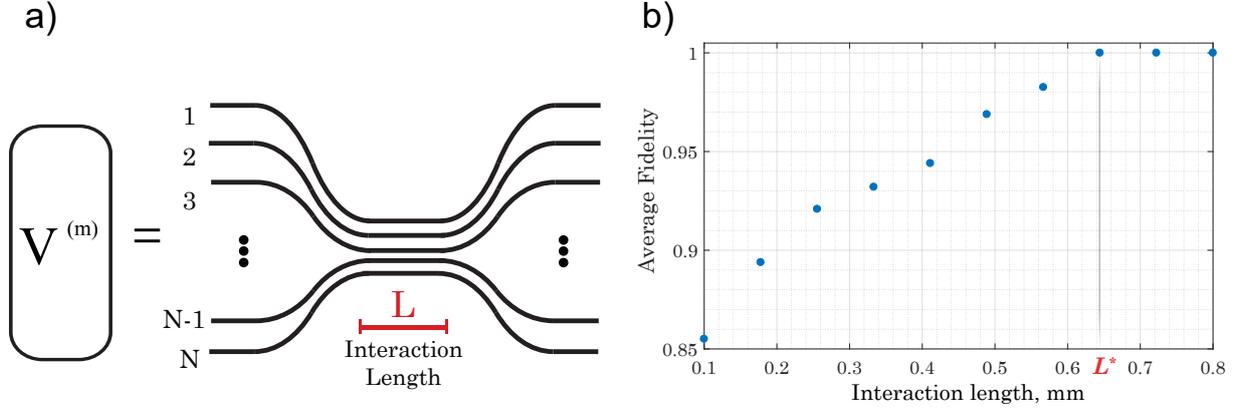


Figure 3. a) Schematic illustration of a linear array of adjacent waveguides used as a mode mixing layer in proposed decomposition (1), depicted in Fig. 1. b) Average fidelity dependence from the mode mixing interaction area length  $L$ . Calculations were performed for  $N = 5$  equally coupled waveguides with coupling constant  $C = 0.5 \text{ mm}^{-1}$ , each point is a mean fidelity over 40 sample random unitary matrices approximation. From certain interaction length  $L^* \approx 0.62$  mm average fidelity of the composer's operation reaches its maximum possible value  $F_{mean} = 1$  with the computational accuracy  $10^{-6}$ .

## 4 CONCLUSION

We have demonstrated new universal multiport interferometer design that may be implemented in linear optics. Our numerical experiments showed that in the proposed unitary composer (1) almost any arbitrary unitary matrix  $V^{(m)}$  may be used as a mode mixing layer without losing the ability to cover the whole  $U(N)$  space, which is a key feature of the proposed multiport interferometer design. We have also proved the feasibility of a linear array of adjacent waveguides to be utilized as a mode mixing layers in the unitary composer. We hope our findings will boost the research in the experimental integrated quantum photonics.

This work has recently been published in [15].

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