The Aperiodic DM-FFF compared to the A-FMM: A Rigorous Method for the Modeling of Guided Optical Structures

Student Paper

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ABSTRACT

The differential method (DM) associated with Fast Fourier Factorization (FFF) has demonstrated its effectiveness with the modeling of metallic periodic diffractive structure especially when the grating is illuminated with TM polarization. In this paper, we will exploit the use of the DM-FFF in the guided optics domain and how this method can be a powerful solution for the design of complex shaped photonic devices.

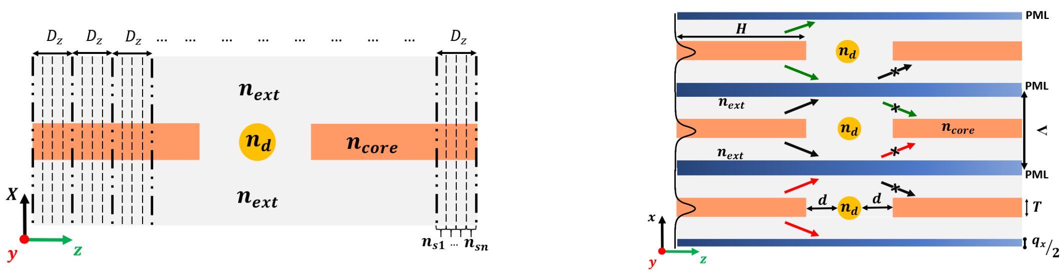
Keywords: Computational Electromagetic, 2D guided optic, Fast Fourier Factorization, Differential Method, AFMM

1 INTRODUCTION

The rapid growth in the domain of integrated optics has forced scientists and researchers to step up their efforts in the development of electromagnetic computational methods that meet with the new available technologies. The differential method (DM) is one of the leading tools for the modeling of diffraction periodic gratings. The association of Fast Fourier Factorization to the DM takes into account the evolution of grating's profile. This new formulation has turned the method into a powerful calculation technique for arbitrary shaped metallic periodic structures especially when the structure is illuminated with TM polarized plane wave. Consequently, stable and accurate results are obtained by eliminating the staircase approximation of the Fourier Modal Method (FMM) [1]. Both methods (DM-FFF and FMM) are known as frequency methods. Therefore, different parameters are expressed in terms of Fourier series by a number of truncated order N following the periodicity axis x.

In their paper [2], Hugonin et al. have demonstrated that performing a complex coordinate transformation to the formulation of Maxwell's equations of the FMM can play the role of perfectly matched layers (PML). This transformation attenuates the incoming and outgoing waves coming from the boundary cells. In this way, the classical FMM will turn into an aperiodic method (A-FMM) used for the modeling of dielectric waveguides structures. Following a similar approach, an implementation of PML with the DM-FFF has been introduced. Moreover, the effectiveness of the A-DM-FFF when it is appplied to arbitrary shaped guided structures compared to the A-FMM will be shown.

2 DM-FFF FROM DIFFRACTIVE OPTICS TO GUIDED OPTICS



(a) (b)

Figure 1: Non linear transform for modeling finite structure from infinite space . The algorithm used discretize the structure into equal layers L_s with a constant D_z discretization step, and for each L_s, n_s sub-layers are used to perform the Runge-Kutta differential integration in case of the A-DM-FFF (a) Infinite isolated space $(X \in [-\infty, +\infty])$ (b) Artificial periodized structure with period Λ bounded in real space $(x \in [0, +\Lambda])$ with the same discretization process.

The propagation equations in Fourier space and the algorithm of the DM-FFF used in diffractive optics is described in Ref.[3]. Hugonin et al. have demonstrated that an open dielectric waveguide structure (waveguide surrounded by semi-infinite homogeneous medium), can be simulated through a periodic structure by exerting a complex coordinate transformation to Maxwell's formulation of the FMM. Following a similar approach and by using the same complex coordinate transformation, the DM-FFF will turn out into an aperiodic method (A-DM-FFF) used in the modeling of arbitrary-shaped guided optical structures. Consequently, the new propagation equation requires the function $f(x) = \left(\frac{\partial F(x)}{\partial x}\right)^{-1}$ that represents the derivative of the non-linear complex transformation function which appears before each $\frac{\partial}{\partial x}$. Therefore, in the Fourier space equation, where K_x is associated to the partial derivative with respect to x, $[f]$ the harmonic vector of the Fourier transformation of $f(x)$ must be added before each K_x. Thus, the new modified propagation formulation that incorporates the complex coordinate transformation can be expressed as,

$$\begin{cases} \frac{\partial [E_x]}{\partial z} = & -j\,[\![f]\!]\,K_x\left([\![k^2]\!] - Q_1\right)^{-1} Q_2 [E_x] \\ & +j\left(I_d - [\![f]\!]\,K_x\left([\![k^2]\!] - Q_1\right)^{-1}\,[\![f]\!]\,[K_x]\right)[H'_y] \\ \frac{\partial [H'_y]}{\partial z} = & j\left(Q_1 + [\![\frac{1}{k^2}]\!]^{-1} - Q_2\left([\![k^2]\!] - Q_1\right)^{-1} Q_2\right)[E_x] \\ & -jQ_2\left([\![k^2]\!] - Q_1\right)^{-1}\,[\![f]\!]\,K_x [H'_y] \end{cases} \tag{1}$$

With, $K_{x,ii} = (-N+i)\frac{2\pi}{\Lambda}$, $[\![A]\!]$ and $[A]$ are the Toeplitz matrix and the Fourier coefficients of the corresponding argument respectively, and $k = \omega\sqrt{\mu\epsilon}$. Q_1 and Q_2 represent the matrices that take the tangential evolution of the field with respect to the studied profile.
This coordinate transformation allows simulating an open structure by using a periodic structure as shown in Fig.1.

3 NUMERICAL RESULTS WITH A CURVILINEAR STRUCTURES

Our method has been validated through a two dimensional rectangular structure studied in ref.[2]. The A-FMM suffers from slow convergence in TM polarization in two cases, 1) in case of high contrast index continuous dielectric profiles, 2) when the method deals with metallic continuous profiles (sinusoidal, trapezoidal, cylindrical,...). Therefore, to appraise the performance of the FFF, a cylindrical reflector of refractive index n_d is placed at the center of the structure. For simplicity reasons, the structure is considered symmetrical. The equations describing the tangential components of the pillar profile are described in Eq.B5 and Eq.B6 of Ref.[1]. We aim to study the structure depicted in Fig.1(a). With $d = 0.15\,\mu m$ and a pillar diameter of $300\,nm$, the structure is illuminated with its fundamental TM_0 guided mode at $\lambda = 1.5\ \mu m$. The period of the unit cell is $\Lambda = 1.1\ \mu m$ and $q_x = 0.4875\ \mu m$, and $n_{core} = 3.5$, the geometry is discretized in the propagation direction z with $D_z = 5\ nm$ and 4 n_s sub-layers are used in the S-matrix propagation of the A-DM-FFF. Comparisons of the convergence rate have been done between the A-DM-FFF and the A-FMM to show the impact of suppressing the staircase approximation of non-lamellar structures.

3.1 Dielectric High Contrast Index Pillar

To possess the high contrast index condition, the guided zone of Fig.1 is surrounded by $n_{ext} = 1.0$ and the dielectric pillar of refractive index $n_d = 3.5$ is chosen. Despite the strong discretization step, the A-FMM converges slowly following the number of harmonics N (blue open circles of Fig.2). This slow convergence is a consequence of the staircase approximation and the parasitic points effect that appears with the high contrast index structures.

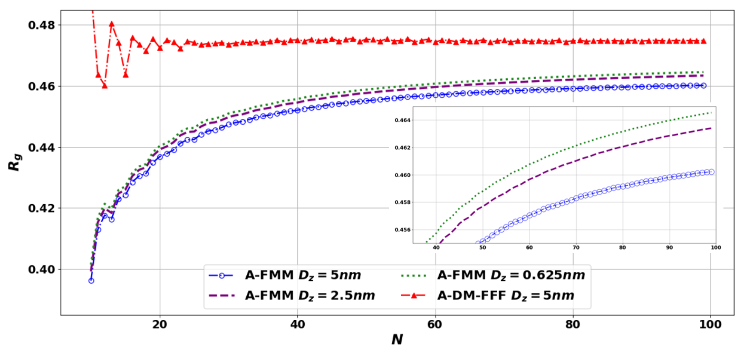


Figure 2: The convergence of the reflection coefficient R_g following N for the high contrast index dielectric structure with dielectric pillar reflector.

Besides, the zoomed part in Fig.2 illustrates that decreasing D_z for the A-FMM from $5\ nm$ to $0.625\ nm$, doesn't enhance the convergence speed, but bring out slowly the result at $N = 100$ to the value of the A-DM-

FFF. On the other hand, with $D_z = 5nm$, the A-DM-FFF (solid red triangles) converges rapidly to a stable value along the evolution of N.

3.2 Metallic Pillar

The same optogeometrical parameters of section 3.1 are considered with a pillar of refractive index $n_d = 1.0 + j7.0$ and $n_{ext} = 2.9$. In Fig.3, the convergence rates following N for each method are reported. The computed intensities R_g of the A-FMM converge slowly as the truncation order N increases. Although, the A-DM-FFF needs $N > 40$ to converge and stabilize following N.

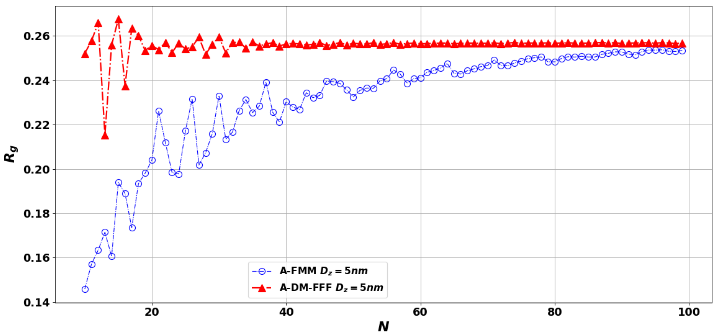
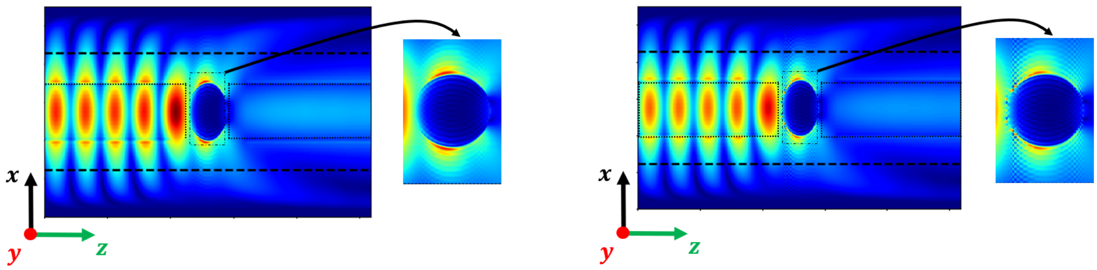


Figure 3: The convergence of the reflection coefficient R_g following N for the dielectric guided structure with metallic pillar reflector.

To reveal the problem induced by the metallic non-lamellar profiles on the field representation, the evolution of $|E_x|$ has been studied for each method with $N = 50$ harmonics. Due to the staircase approximation, parasitic reflections appear at the dielectric-metal interface of the pillar with the A-FMM (Fig.4.b). While in case of the A-DM-FFF, the problem is mostly treated and the parasitic interference of the field is suppressed (Fig.4.a).



(a) (b)

Figure 4: Evolution of $|E_x|$ (a) With the A-DM-FFF (b)With the A-FMM .

4 CONCLUSION

In this paper, a new formulation that turn the DM-FFF into aperiodic method has been proposed. We proved that the DM-FFF can incredibly enhance the convergence for guided non-Lamellar structures in case of dielectric high contrast index structure and metallic structures. We believe that this method will open the way for researchers and engineers to simulate and model new arbitrary shaped guided structure that was considered as problematic before.

ACKNOWLEDGMENT

The authors thank the French National Agency of Research (ANR) for funding. This work was supported by the ODISSEA project under Grant No.16-CE39-0016-01

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