Bifurcation and Chaotic Behavior of Feedback Semiconductor Lasers Operating in the Full Range of External Reflectivity

Qin Zou, Kamel Merghem, Badr-Eddine Benkelfat, Musharrat Shabnam

Samovar, CNRS, Télécom SudParis, Institut Polytechnique de Paris,
19 place Marguerite Perey, 91120 Palaiseau, France
e-mail: qin.zou@telecom-sudparis.eu

ABSTRACT

We investigate the properties of a semiconductor laser with arbitrary levels of external optical feedback, by means of an iterative travelling-wave (ITW) model. Bifurcation diagrams and the corresponding Lyapunov exponent are used to describe the behavior of the feedback laser as the external feedback reflectivity increases from 0 to 1. Similarities and differences between the ITW model and the Lang-Kobayashi (LK) model are also reviewed. To determine the critical value of the external reflectivity, beyond which the LK model would no longer be valid, we propose a criterion based on the iterates of the phase maps for both the compound-cavity modes (ITW model) and the external-cavity modes (LK model). Preliminary results show that this critical value may correspond to the so-called «r_∞-value», which is defined as the onset value of the first chaotic region.

Keywords: Semiconductor lasers; External optical feedback; Lang-Kobayashi model; Iterative travelling-wave model; Bifurcations and deterministic chaos; Lyapunov exponent.

1. INTRODUCTION

A semiconductor laser with delayed optical feedback builds an ideal system for analysing and exploring typical phenomena observed in a nonlinear time-delayed system, such as bifurcations, periodic oscillations, onsets of instability, and chaotic itinerancy. In order to describe adequately these phenomena, often occurring in the moderate- or strong-feedback regime of operation, an iterative travelling-wave (ITW) model was developed [1]. Here we summarize briefly our recent work on building, with this model, the bifurcation route of a feedback laser with the whole interval [0, 1] of external reflectivity (regimes I-V [1]) and present some preliminary results.

2. ITERATIVE TRAVELLING−WAVE MODEL

Consider the configuration of Figure 1. A single-longitudinal-mode laser diode is in resonance with an external Fabry-Perot cavity. We assume that r_1, r_2, and r_3 are all real and dispersionless. For this three-mirror-coupled system, multiple round trips inside the external cavity should be in general taken into account for an arbitrary feedback level, namely for any value of r_3 between 0 and 1.

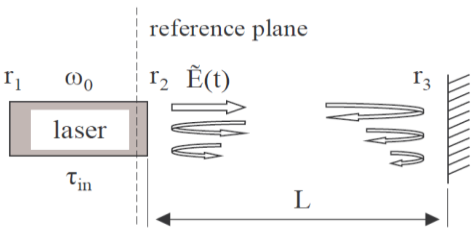

Figure 1. Schematic of a single-mode laser diode with external optical feedback. ω_0: emission (angular) frequency of the solitary laser; τ_{in}: internal round-trip time; r_1: reflection coefficient of the rear facet of the laser; r_2: reflection coefficient of the front facet of the laser; r_3: reflection coefficient of the external mirror; $\tilde{E}(t)$: right-moving electrical field; L: length of external cavity assumed empty.

2.1 Steady-state solutions

It has been shown [1] that the right-moving electrical field $\tilde{E}(t)$, which is calculated at steps of the internal round-trip time τ_{in} (in seconds), satisfies the following iterative equation:

$$\tilde{E}(t+\tau_{in}) = \frac{1}{r_2^2} exp\left\{\frac{\tau_{in}G_N}{2}(1+i\alpha)[N(t)-N_{th}]\right\}$$
$$\times\left[\tilde{E}(t) - (1-r_2^2)\sum_{m=0}^{M}(-r_2r_3)^m\tilde{E}(t-m\tau)exp(-im\Delta_0)\right] \quad (1)$$

In this equation, G_N (in $s^{-1}m^3$) is the differential gain; α is the linewidth enhancement factor; $N(t)$ (in m^{-3}) is the carrier density and N_{th} its threshold; τ (in seconds) is the external round-trip time and Δ_0 (in radian, $\Delta_0 = \omega_0\tau$) is denoted as the initial feedback phase which is associated with the emission (angular) frequency ω_0 of the solitary laser operating just above threshold.

By inserting $\tilde{E}(t) = \tilde{E}_0(t)exp(i\omega t)$ into Equation (1) and considering steady-state solutions for a feedback-induced compound-cavity mode, we obtain the following expressions for the excess gain δG (in s^{-1}) and the feedback phase Δ (in radian, $\Delta = \omega\tau$, with ω: possible emission frequency):

$$\delta G = \frac{-1}{\tau_{in}} ln \frac{(1-D)^2+E^2}{r_2^4} \tag{2}$$

$$tg\left(b_2 - \Delta\frac{\tau_{in}}{\tau}\right) + \frac{E}{1-D} = 0 \tag{3}$$

In these two equations, D (dimensionless), E (dimensionless), and b_2 (in radian) are written respectively as:

$$D = (1 - r_2^2)\sum_{m=0}^{M}(-r_2r_3)^m cos[m(\Delta_0 + \Delta)] \tag{4}$$

$$E = (1 - r_2^2)\sum_{m=0}^{M}(-r_2r_3)^m sin[m(\Delta_0 + \Delta)] \tag{5}$$

$$b_2 = \frac{-\alpha}{2} ln \frac{(1-D)^2+E^2}{r_2^4} \tag{6}$$

2.2 Bifurcation diagram

In Figure 2, is illustrated an example of the accumulation points of the iterates, whose values are calculated from the transcendental phase equation, Equation (3), for the full variation range (from 0 to 1) of r_3. The reflection coefficient of the front facet is taken as $r_2 = 0.65$ (HR-coated facet). The other parameter values are: $r_1 = \sqrt{0.3}$ (as-cleaved facet), $\alpha = 5$, $\tau_{in} = 9 \times 10^{-12}$ (s), $\tau = 9 \times 10^{-10}$ (s), and $M = 50$ (round-trip number).

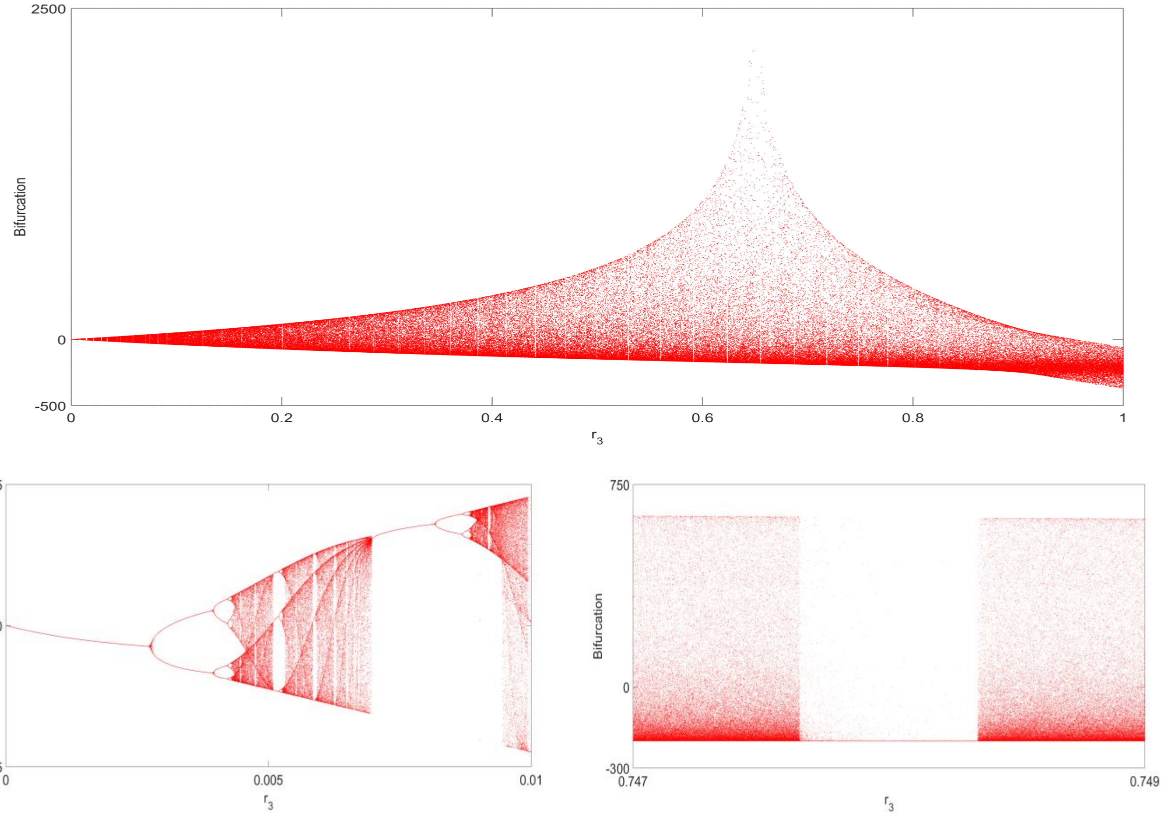

Figure 2. Bifurcation diagram over the whole interval [0, 1] of r_3, where the chaotic regime is interrupted by r_3-windows (upper); Zoom on the region corresponding to small values of r_3 (moderate-feedback regime, where the LK model may still be valid) (bottom-left); A narrow window between two adjacent chaotic regions in the strong-feedback regime (where the use of the LK model is no longer justifiable). This behavior confirms previous experimental observations: in the moderate- or strong-feedback regime, the system could be re-stabilized and as a result stable single-mode operation could be expected with r_3 close to 1 (bottom-right).

3. COMPARISON BETWEEN THE ITW MODEL AND THE LK MODEL

In a previous study [2], we showed that in the weak-feedback regime ($r_3 \ll 1$, regime I), the two models are identical if (and only if) $\Delta_0 = 0$, and that any stationary solution to Equation (1) will lead to the following equation for both two models:

$$\frac{G_N}{2}(1 + i\alpha)(N_s - N_{th}) + \gamma exp(-i\Delta) - i\omega = 0 \quad (7)$$

where N_s is the steady-state carrier density and γ (in s^{-1}) is the feedback rate which is defined as usual: $\gamma = (1 - r_2^2) r_3 r_2^{-1} \tau_{in}^{-1}$. Equation (3) will then reduce to the widely-employed phase condition [3]:

$$\Delta = -\gamma\tau(\alpha \cdot cos\Delta + sin\Delta) \quad (8)$$

More generally, to determine the limit of the LK model, we compare the iterates of the phase maps obtained respectively from Equations (3) and (8). Figure 3 illustrates the difference between these two iterates, for the value of r_3 varying from 0 to 0.0045 (moderate-feedback regime). This figure suggests a critical value of about 0.002984 for r_3. This value might be (at least is very close to) the onset value (r_∞-value) of the first chaotic region, at which the number of fixed points becomes infinite and the Lyapunov exponent is also equal to zero.

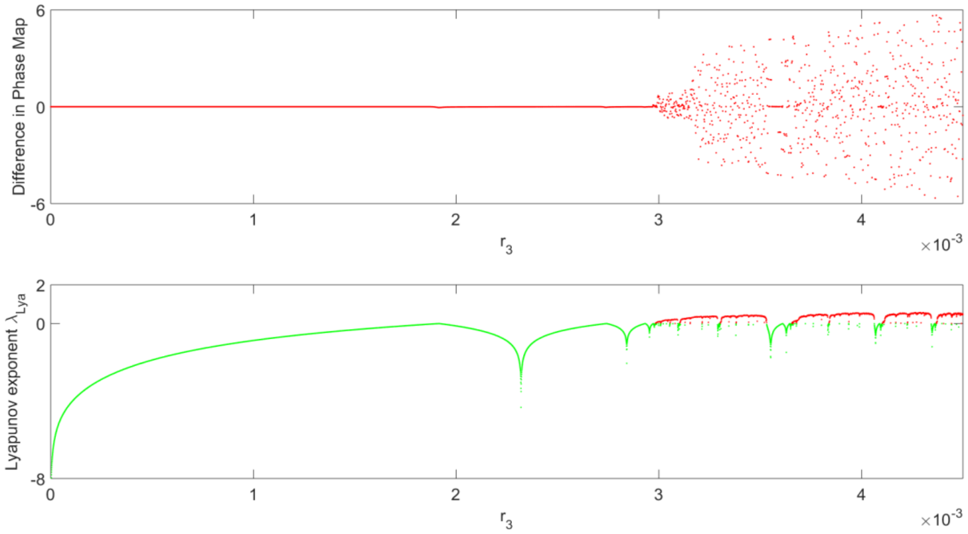

Figure 3. Difference in phase map between the ITW model and the LK model (upper); Lyapunov exponent λ_{Lya} calculated from Equation (3). A bifurcation takes place at $\lambda_{Lya} = 0$. Chaotic behavior corresponds to $\lambda_{Lya} > 0$ (red dots) (bottom).

4. CONCLUSIONS

The iterates of the phase map and the Lyapunov exponent determine without ambiguity a state of a feedback laser and provide therefore a clear description of its behavior in each of the five feedback regimes so far classified, such as super-stable states, bifurcation points, periodic and chaotic regions, and re-(in)stabilization onsets. Stable single-mode operation could be expected in the strong-feedback regime due to the opening of reflectivity-windows inside the chaotic regions. The reflectivity value corresponding to the onset of the first chaotic region may be considered as the critical value for identifying the limit of the Lang-Kobayashi model. Future work will include an experimental investigation of a laser operating in the strong-feedback regime.

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REFERENCES

[1] L. N. Langley, K. A. Shore, J. Mørk: Dynamical and noise properties of laser diodes subject to strong optical feedback, *Optics Letters*, vol. 19, No. 24, pp. 2137-2139, 1994.

[2] Q. Zou: Steady-state behavior of semiconductor laser diodes subject to arbitrary levels of external optical feedback, *Optics and Photonics Journal*, vol. 3, No. 1, pp. 128-134, 2013.

[3] R. Lang, K. Kobayashi: External optical feedback effects on semiconductor injection laser properties, *IEEE Journal of Quantum Electronics*, vol. 16, No. 3, pp. 347-355, 1980.