Recursive MZI mesh for integral equation implementation

(Student Paper)

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ABSTRACT

In this work we present a photonic integrated circuit based on a mesh of Mach-Zehnder Interferometers (MZIs) acting as a programmable photonic equation solver. The working principle is demonstrated by numerical simulations showing all-optical solution of a Fredholm integral equation. A thermally actuated circuit is fabricated on a silicon platform demonstrating the feasibility and programmability of the proposed device concept. Circuit calibration has been experimentally demonstrated and validated by simulations.

Keywords: silicon photonics, reconfigurable PIC, Fredholm integral equations, optical analogue computing

1. INTRODUCTION

Lately, there has been an increasing interest in programmable photonic integrated circuits (PICs) that can be reconfigured during operation to perform different linear functions. For instance, meshes of Mach-Zehnder interferometers (MZIs) [1] have been successfully used in a wide range of applications, such as mode-division multiplexing and demultiplexing, imaging, beam steering, quantum information processing and deep learning.

In this work, we exploit programmable meshes of silicon MZIs to bring to photonics the concept of an equation solver that has been recently demonstrated in the microwave domain by using a suitably engineered metamaterial [2,3]. With respect to this pioneering demonstration, photonics offers the benefits of footprint reduction, higher scalability to more complex systems of equations, programmability and wider bandwidth. As a proof-of-concept demonstrator, we conceived a recursive photonic circuit made of a 3x3 mesh of silicon MZIs enclosed in an array of optical feedback loops, which can be thermally programmed to solve systems of Fredholm integral equations of the second kind with arbitrary unitary kernels.

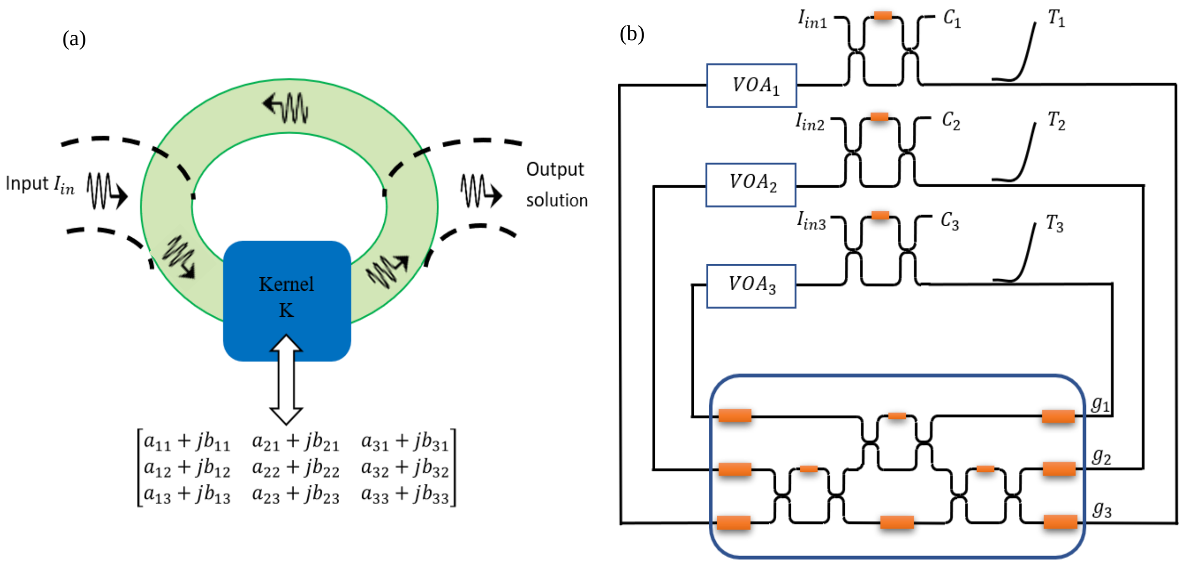

Figure 1: Programmable photonic equation solver: (a) schematic concept of the device and (b) circuit topology of the PIC. A mesh of tunable couplers implementing the programmable kernel K is enclosed in a feedback loop array of optical waveguides.

2. RECURSIVE MZI MESH

Figure 1 shows the schematic concept of the proposed equation solver PIC (a) together with its photonic circuit topology (b). A triangular mesh of MZI is used to implement a programmable kernel (K) and a feedback network of optical waveguides is used to make the light recirculate through the kernel. This scheme recalls the recursive solution of a Fredholm integral equation of second kind as expressed in equation (1):

$$g(x,t) = I_{in}(x,t) + \int K(x,y)\, g(y,t)\, dy. \quad (1)$$

where $I_{in}(x,t)$ is the vector containing the complex amplitudes of the optical field at the input ports and $g(x,t)$ are the optical fields at the beginning of the kernel. In the considered 3x3 mesh implementation, x and y take discrete integer values {1, 2, 3} labelling the corresponding waveguide loops. Each optical input $I_{in}(x,t)$ is coupled to the feedback loops by means of tunable couplers realized by using balanced MZIs. The kernel is composed of three MZIs each with two phase shifters for the implementation of amplitude- and phase-tunable couplers. Additional phase actuators are used to adjust the round-trip phase of each loop. Therefore, nine phase-shifters are enough to implement any unitary matrix of arbitrary complex amplitudes. Variable optical attenuators (VOAs) are also integrated in the feedback loops to either compensate for differential loss or to provide additional degrees of freedom for PIC tuning. It should be noted that, since the optical inputs need to be injected in the device through optical couplers, the optical response of the circuits approaches the integral equation (1) only when these coupling ratios are reasonably small (typically < 10%).
As an example, consider a kernel implementing a frequency domain Fourier-based tritter matrix [4] as expressed in equation (2):

$$K = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & e^{j2\pi} \\ e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} & e^{j4\pi} \\ e^{j2\pi} & e^{j4\pi} & e^{j6\pi} \end{bmatrix} \quad (2)$$

The solution *g(x,t)* of equation (1) is provided by the complex-amplitude distribution of optical waves at the input of the kernel or equivalently by their frequency domain expression *g(x,f)*. The information is then extracted by 10% taps. Figure 2 shows the comparison between the intensity (a) and phase (b) of *g(x,f)* as calculated by numerically solving equation (1) (solid lines) and by simulating the optical response of the PIC of Fig. 1b. In the considered case the optical inputs are set to $I_{in}(x,f) = [0\ \ 0\ \ 1]$ at any wavelength, the input tunable couplers are 10% and all the feedback loops share the same round trip phase. Due to the presence of three loops, the frequency domain response exhibits three resonance peaks, whose wavelength position is given by the phase terms of the kernel. By selecting a specific wavelength (e.g. λ = 1556.909 nm), the steady state solution to a CW input vector is found, as shown in Fig. 2c.

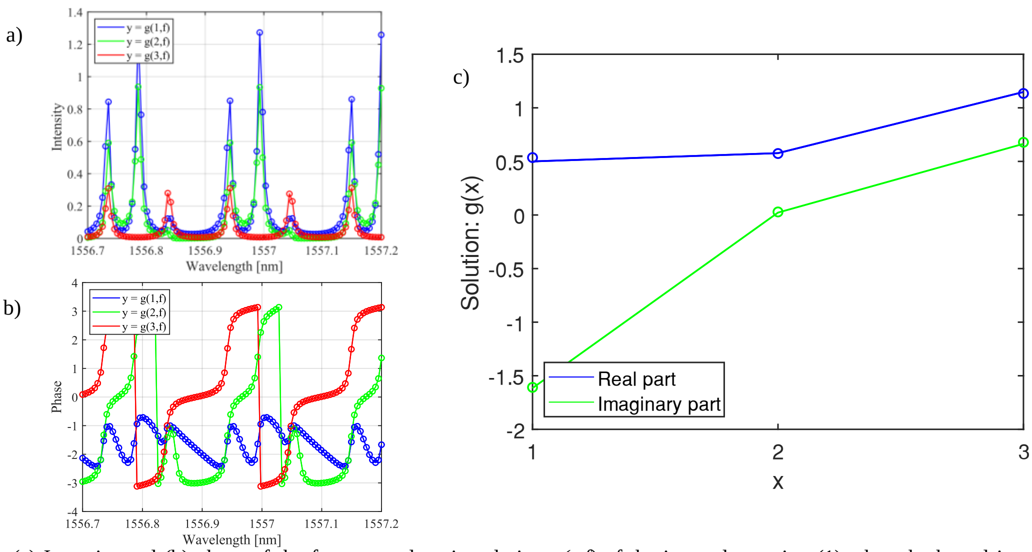

Figure 2: (a) Intensity and (b) phase of the frequency-domain solution *g(x,f)* of the integral equation (1) when the kernel is set to a Fourier-based tritter matrix, all the non-linearities considered. Comparison between the analytical solution (solid lines) and numerical simulation of the photonic equation solver (circles). (c) Comparison between the real (blue) and imaginary (green) part of the analytical solution (solid line) and of the numerical simulation of the photonic equation solver (circles) at a wavelength of 1556.909 nm.

3. EXPERIMENTAL RESULTS AND KERNEL CALIBRATION

A proof-of-concept demonstrator of the programmable photonic equation solver was realized on a standard 220 nm silicon photonic platform (AMF, Singapore). The tunable couplers of the 3x3 programmable kernel, as well as those used to couple the optical inputs to the PIC, are realized by means of thermally tunable balanced MZIs. The feedback loops share the same physical length of 2786.869 μm, providing a free spectral range of 0.214 nm, and the overall device footprint is 1.1 x 1.48 mm^2. Figure 3a shows a top-view picture of the photonic chip assembled on an electronic printed circuit board (PCB) hosting the drivers of the thermal tuners.

Preliminary tests were carried out to demonstrate the possibility to program the kernels and bring the PIC to the desired working point. Targeting the implementation of the tritter matrix of equation (2), a specific algorithm was developed to set the thermal tuners of the 3x3 mesh to the required phase shift. Implementing a three-way beamsplitter matrix means splitting equally the optical power at the three outputs, therefore, the upper interferometer of the mesh must be set at 67% split ratio while the other two MZIs should work as 50% splitters. The implemented calibration procedure implies a sequential injection of the light from the tunable MZIs (I_{in1} - I_{in3} ports) and the acquisition of the optical spectrum from the output ports (C_1 - C_3). Once the desired intensity response of the mesh is achieved, the phase response must be adjusted, also taking into consideration phase contribution given by fabrication tolerances. In order to have robust phase control, strategies to mitigate thermal crosstalk among thermal tuners can be also adopted [5].

Figure 3 shows the simulated (b) and measured (c) frequency domain response of the device when the 3x3 mesh operates as a three-way beamsplitter and all the feedback loops are set to resonate at the same frequency. Each curve in the plot corresponds to a different input (I_i) - output (C_i) pair. From this condition, the phases of the kernel elements can be adjusted by exploiting the feedback loops as integrated interferometers, mapping phase shifts to resonance wavelength shift. For instance, Fig. 3d shows the effect of an additional $2\pi/3$ shift in the elements K_{11} and K_{22}, resulting in a wavelength shift of FSR/3, that is 0.071 nm. The agreement between the expected and the observed behaviour demonstrates the possibility of programming the mesh to any arbitrarily complex kernel.

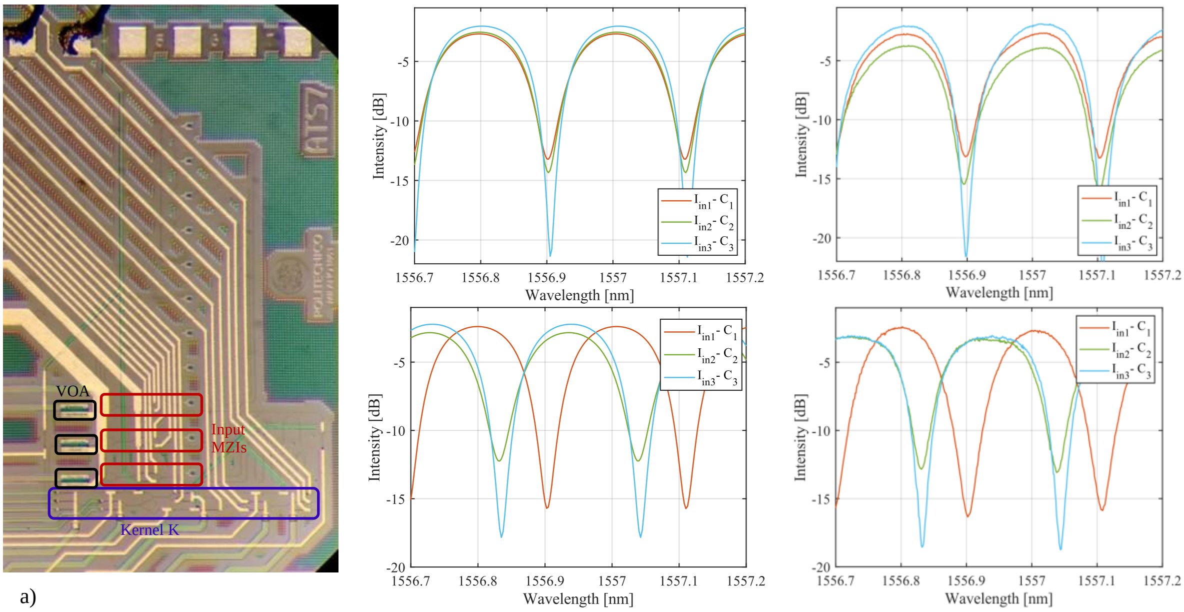

Figure 3: (a) Top view photograph of our bonded chip mounted on its specific PCB. (b) Optical simulation results of the spectrum collected at the MZIs port (C_1-C_3) when the mesh is configured (in amplitude) as a tritter matrix, so that the power is split equally in the three output waveguides. (c) Experimental results when the mesh represents the three-way beamsplitter matrix. See comparison with (b). (d) Optical simulation results of the spectrum collected at the MZIs port (C_1-C_3) port when the main diagonal phases of the tritter matrix are set. (e) Experimental results when also the main diagonal phases are considered. See comparison with (c).

4. CONCLUSION

We developed a programmable photonic integrated equation solver based on meshes of MZI. This concept is used to solve a Fourier-based tritter matrix and results are compared with analytical solutions. We presented precise calibration of the fabricated circuit to implement a desired transformation and results are validated by simulation. Experimental activities for implementation of specific integral equation are ongoing.

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