

# Broad-band phase-matched multimode waveguides for application to four-wave mixing

(Student Paper)

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## ABSTRACT

We present a general method to equalize the frequency spacings for application to inter-modal degenerate four-wave mixing in a multimode waveguide. By applying the proposed method, the dispersion of the modes can be controlled allowing flexible phase to frequency and wave vector phase matching (PM) between several propagative modes. Taking advantage of the proposed method, PM can be satisfied over a wide wavelength range, e.g. bridging from telecom wavelength to almost 2  $\mu\text{m}$ . In addition, the reported approach releases new degrees of freedom to tune group velocity dispersion to achieve four-wave mixing in single-mode waveguides for on different material platforms and waveguide thicknesses. We believe that this method has an immense potential for a plethora of all-optical signal processing applications in the telecom range, mid-infrared light generation and Brillouin scattering with selectable phonon energy.

**Keywords:** Phase-matching, nonlinear processes, dispersion, multi-mode operation, broadband conversion, self-adaptive boundary.

## 1. INTRODUCTION

Nonlinear processes are of paramount relevance for on-chip signal processing and spectroscopy in the near- and mid-infrared [1]. Functions like optical parametric amplification based on the Kerr effect, have been demonstrated [2] on the mainstream silicon (silicon, silicon nitride, silicon/germanium compound) platform. Among these works, compensating the intrinsically material dispersion and the nonlinearity-induced dispersion, plays a key role [3], especially when considering the four-wave mixing processes (FWM). To tackle this phase matching (PM) problem, different waveguides with optimized dimensions have been proposed, still showing a limited control of the dispersion profile. Figure 1(a) shows phase and energy matching conditions as well as typical dispersion profile in conventional FWM based on single-mode operation.

In this context, we focus on multimode waveguides for the realization of phase matching with a wavelength span up to few hundreds of nanometers. We consider 3 waveguide modes. We present a novel method for facilitating FWM in multimode graded-profile waveguides. By introducing a “self-adaptive boundary” approach, we simultaneously satisfy energy conservation and wavevector matching multimode waveguides for nonlinear purposes, see Fig. 1(b). This approach obviates the requirement for an exact dispersion by ensuring an equal frequency spacing between modes in a remarkably wide propagation wavevector range.

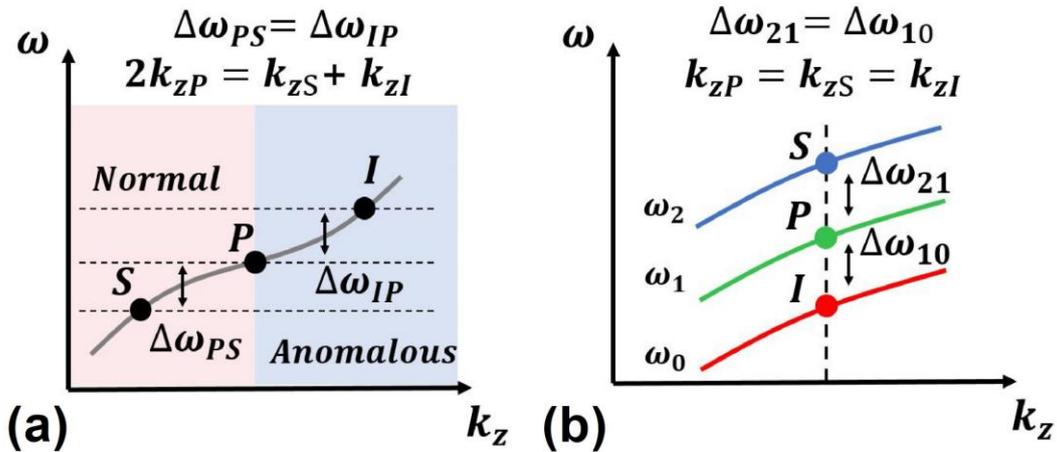


Figure 1. Schematic of (a) degenerate four wave mixing operating in a single-mode waveguide with anomalous dispersion and (b) operating within an inter-modal scheme regardless the absolute dispersion provided that all dispersion curves are obtained by translating the same curve with a constant frequency step (e.g.  $\Delta\omega_{21} = \Delta\omega_{10}$  here).

## 2. COMPARISONS AND RESULTS

As it is well known, in a two-dimension step-index slab waveguide (infinite depth along the  $x$  axis), the eigen equation for modes propagating along the  $z$  axis with an electric field polarized along  $y$  axis reduces to [4]:

$$ha = \frac{m\pi}{2} + \arctan\left(\frac{\gamma n_w^2}{h n_c^2}\right) \quad (1)$$

in which  $n_w$  and  $n_c$  is the material index of waveguide core and cladding while  $h = \sqrt{(k_0^2 n_w^2 - k_z^2)}$  and  $\gamma = \sqrt{(k_z^2 - k_0^2 n_c^2)}$  are the wavevector along the  $y$  axis, inside and outside the waveguide core, respectively. The dispersion curves from solution equation (1) for the first four order modes (with corresponding mode order  $m=0, 1, 2, 3$ ), are shown in the inset of Fig. 2(a) with the index profile and the mode profiles of the first three order mode depicted at the bottom-right corner. Silicon and silicon dioxide are chosen here as the materials (material dispersion was considered) while the waveguide width  $2a$  was set at 700 nm. Clearly for a fixed wavevector  $k_z$  (tracked at  $k_z = 1.08 \times 10^7 \text{ m}^{-1}$  here), the frequency spacing between mode  $m$  and  $m+1$  increases sharply with  $m$ , hampering the degenerate FWM process. By conducting the derivative of equation (1) to  $n_{eff}$ , we can obtain an analytical frequency spacing as:

$$\frac{d\omega}{dm} = \frac{d}{dn_{eff}} \left( \frac{k_z c}{n_{eff}} \right) / \frac{dm}{dn_{eff}} \approx \frac{c\pi \sqrt{n_w^2 - n_{eff}^2}}{2an_w^2} \quad (2)$$

Using this equation (2), we represent the analytically continuous frequency spacing  $\frac{d\omega}{dm}$  for a waveguide with  $a=350$  nm,  $n_w=3.48$ ,  $n_c=1.445$ , as in Fig. 2(b). The effective index  $n_{eff}$  of the first 4 modes are marked by the circles in the figure,  $\frac{d\omega}{dm}$  at which are well coincident with the discrete calculation results coming from the dispersion curves shown in Fig. 2(a). This confirms that the analytical frequency spacing  $\frac{d\omega}{dm}$  can be considered as an effective figure of merit to investigate how the frequency spacing evolves with the waveguide dimension and the index profile. From Fig. 2(b), we notice that the frequency spacing is almost proportional to  $\sqrt{n_w^2 - n_{eff}^2}$ , in the usual range of  $n_{eff}$  (1.6 to 3.3, for silicon waveguide cladded by silica). This comes from the reciprocal relationship between  $\omega$  and  $n_{eff}$  for an unchanged  $k_z$ , on the left part of equation (1), compared to the almost linear phase increment with  $m$  on the right.

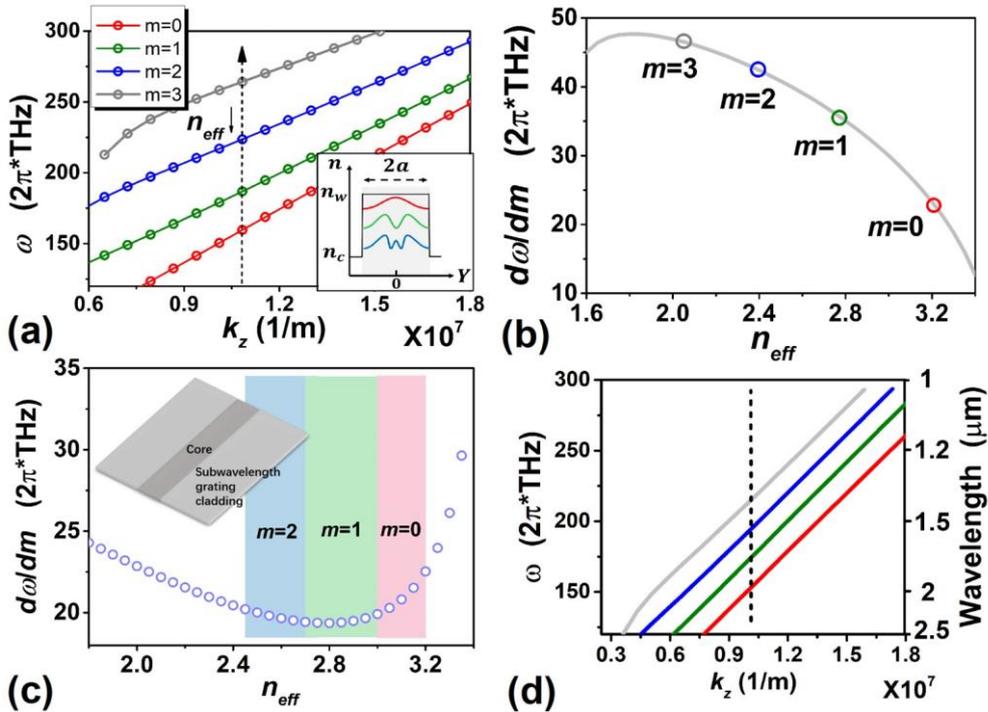


Figure 2. (a) Dispersion curves of first 4 modes of a two-dimension silicon waveguide with silica cladding, propagating along  $z$  axis. The width is  $2a = 700$  nm. The index profile of the waveguide and mode distribution is plotted in the inset. (b) Analytical frequency spacings as a function of effective index  $n_{eff}$  of the waveguide in (a). (c) Analytical Frequency spacings as a function of effective index  $n_{eff}$ , of a waveguide using self-adaptive boundary. The  $n_{eff}$  points, corresponding to the first 3 modes, are labelled by the color regions. (d) Dispersion curves of first 4 modes with self-adaptive boundary, calculated using 3D FDTD.

To overcome this limitation, we propose to implement a subwavelength engineered waveguide lateral cladding (see top-left of Fig. 2(c)) that enables the implementation of a tailorable index profile. By introducing a scenario inherited from potential wells and adopting a well-designed index profile, we are able to create an effective photon well along the transverse direction (perpendicular to the wave-propagation direction) for fine tuning the waveguide dispersion.

The optimized waveguide has  $a=800$  nm,  $n_w=3.48$ ,  $n_c=1.45$  and a well-designed index profile. Frequency spacings as a function of  $n_{eff}$  using equation (3) are plotted in Fig. 2(c). The frequency spacings change no longer monotonously, instead, a new tendency is created with a local minimum ( $n_{eff} \approx 2.8$ ). The index region of the first 3 modes are labelled by colour regions which is right around the turning point of the curve and indicating the possibility of identical frequency spacing.

To validate these theoretical predictions, we conducted 3D FDTD calculation based on the parameters used in Fig. 2(c) and displayed the corresponding dispersion curves of the first 4 order modes in Fig. 2(d). Clearly, all the curves are almost parallel and separated by a very a close frequency spacing of around 20.5 THz, which is perfectly coincident to the prediction indicated by the analytical calculation.

### 3. CONCLUSION

We propose a new approach for designing multimode graded waveguides capable of reshuffling the needed conditions (simultaneous satisfaction of the energy conservation and wavevector phase matching) for the exploitation of degenerate FWM, offering a flexible strategy for versatile material platforms regardless the thickness and intrinsic dispersion of the considered optical waveguide modes. We provide a thorough analytical explanation and numerical confirmation of the theoretical predictions. The proposed approach can be scaled up (e.g. 5 modes or 7 modes), with the possibility of achieving phase-matching for wavelength ranges over a few hundreds of nanometers, which we believe will benefit nonlinear applications, e.g. light generation in mid-infrared wavelength and applications in which dispersion manipulation is of first priority.

### ACKNOWLEDGEMENTS

The French ANR agency is acknowledged for the its support through the SITQOM project and MIRSPEC project (ANR-17-CE09-0041)

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