# Distributed Phase Shift and Lasing Wavelength in Distributed-Feedback Resonators

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## **ABSTRACT**

Distributed-feedback waveguide lasers based on Bragg-grating resonators generate ultranarrow-linewidth emission. The required  $\pi/2$  phase shift is often introduced by a distributed change in effective refractive index, e.g. by adiabatically widening the waveguide. Despite careful design and fabrication, the experimentally observed resonance wavelength deviates, thereby placing the resonance and laser emission at a position with lower reflectivity inside the reflection band. This effect is often incorrectly attributed to fabrication errors. Here we show theoretically and experimentally that the decay of light intensity during propagation from the phase-shift center into both sides of the Bragg grating due to (i) reflection by the periodic grating and (ii) the adiabatic refractive-index change causes an incomplete accumulation of designed phase shift, thereby systematically shifting the resonance to a shorter wavelength. Calculations based on the characteristic-matrix approach and experimental studies in a distributed-feedback channel-waveguide resonator in amorphous  $Al_2O_3$  on silicon with a distributed phase shift introduced by adiabatic widening of the waveguide according to a  $\sin^2$  function show good agreement.

Keywords: Integrated Lasers, Distributed-feedback lasers, Optical resonators, Laser resonators, Bragg reflectors.

## 1. INTRODUCTION

Distributed-feedback (DFB) lasers [1, 2] utilize Fabry-Pérot-type resonators [3] with distributed mirror reflection and exploit the spectral selectivity of periodic Bragg gratings to generate ultranarrow-linewidth emission for various applications. An additional phase shift  $\delta\phi_{design}$  of  $\pi/2$  is required to design a resonance wavelength  $\lambda_{design}$  located at the Bragg wavelength  $\lambda_B$ . A common fabrication method inscribes a laser interference pattern into a photosensitive guiding region or into a cladding [2, 4], and a distributed phase shift, e.g. by widening the guiding region, increases the effective refractive index [5, 6]. Often the experimental resonance deviates from the designed wavelength because of fabrication inaccuracies or a chirp of the Bragg grating, induced thermally by optically pumping a DFB laser from one waveguide end [7]. Hillmer et al. [8] introduced a chirp, with a designed phase shift of  $\pi/2$  distributed over different lengths, and showed that the longer this length, the more the resonance is shifted towards shorter wavelengths. Here we investigate a different effect. Our investigation [9] concerns DFB resonators, where the light intensity extends over a length comparable to the distributed-phase-shift region. Due to the distributed reflection, the intensity of light propagating from the phase-shift center in both directions into the grating decreases, depending on the grating coupling coefficient and the effective-refractive-index increase in the phase-shift region. Consequently, the oscillating light does not experience the full designed phase shift, leading to a negative wavelength shift of the resonance peak with respect to the design. We investigate this effect in a situation where it is the key effect responsible for the wavelength shift, not being overshadowed by other effects.

# 2. THEORETICAL INVESTIGATIONS

When the designed phase shift is  $\delta\phi_{design}=\pi/2$ , the resonance wavelength  $\lambda_{design}$  is located at the center of the reflection band. A designed phase shift  $\delta\phi_{design}\neq\pi/2$  places the resonance wavelength  $\lambda_{design}$  at a different spectral location. Both, the reflection bandwidth and the wavelength difference

$$\delta\lambda_{\text{design}} = \lambda_{\text{design}} - \lambda_{\text{B}} \tag{1}$$

increase with the grating coupling coefficient  $\kappa$ . Along the waveguide direction z, the distributed-phase-shift region is centered at position  $z_{ps}$  and extends over a length  $\ell_{ps}$ . In this region, the waveguide is widened from its normal width w to its maximum width at  $z_{ps}$  and then narrowed back to w, with a profile  $\delta w_{ps}(z)$ . The effective refractive index of the waveguide mode increases with the same profile  $\delta n_{ps}(z)$ . The total additional phase shift accumulated by light propagating through the entire phase-shift region with constant intensity would then be given by [10]

$$\delta\phi_{\text{design}} = \frac{2\pi}{\lambda_{\text{B}}} \delta n_{\text{design}} \ell_{\text{ps}} = \frac{2\pi}{\lambda_{\text{B}}} \int_{z_{\text{m}} - \ell_{\text{m}}/2}^{z_{\text{ps}} + \ell_{\text{ps}}/2} \delta n_{\text{ps}}(z) dz, \qquad (2)$$

where  $\delta n_{design}$  is the mean effective-refractive-index change within the phase-shift region. Our waveguide widening  $\delta w_{ps}(z)$  and, hence, the effective refractive index increase  $\delta n_{ps}(z)$  is designed to follow a  $\sin^2$  function [5],

$$\delta n_{\rm ps}(z) = \delta n_{\rm ps}(z_{\rm ps}) \sin^2 \left[ \pi \frac{\left(z + \ell_{\rm ps}/2 - z_{\rm ps}\right)}{\ell_{\rm ps}} \right]. \tag{3}$$

Insertion of equation (3) into equation (2) and integration yields

$$\delta\phi_{\text{design}} = \frac{\pi}{\lambda_{\text{B}}} \delta n_{\text{ps}} \left( z_{\text{ps}} \right) \ell_{\text{ps}} \,. \tag{4}$$

To obtain  $\lambda_{design} = \lambda_B$ , one choses  $\delta n_{ps}(z_{ps})$  and  $\ell_{ps}$  in equation (4) such that  $\delta \phi_{design} = \pi/2$ . However, due to a deviation  $\Delta \phi_{res}$  of the actual distributed phase shift  $\delta \phi_{res}$  from the designed value of  $\delta \phi_{design}$ ,

$$\Delta\phi_{\rm res} = \delta\phi_{\rm res} - \delta\phi_{\rm design} \,, \tag{5}$$

the actually obtained resonance wavelength  $\lambda_{res}$  is typically shorter than  $\lambda_{design}$  by

$$\Delta \lambda_{\rm res} = \lambda_{\rm res} - \lambda_{\rm design} \implies \lambda_{\rm res} - \lambda_{\rm B} = \delta \lambda_{\rm design} + \Delta \lambda_{\rm res} , \qquad (6)$$

resulting in a total wavelength shift from the Bragg wavelength of  $\delta \lambda_{design} + \Delta \lambda_{res}$ . This additional deviation  $\Delta \lambda_{res}$  in a distributed-phase-shift DBF resonator is due to an incomplete accumulation of phase shift by the light intensity I(z) that varies with position z. Consequently,  $\delta \phi_{res}$  is smaller than  $\delta \phi_{design}$  and amounts to

$$\delta\phi_{\rm res} = \frac{1}{I_{\rm max}} \frac{2\pi}{\lambda_{\rm B}} \sum_{z_{\rm ps}-\ell_{\rm ps}/2}^{z_{\rm ps}+\ell_{\rm ps}/2} \delta n_{\rm ps}(z) I(z) dz.$$
 (7)

This expression determines the accumulated phase shift and the resonance wavelength within the reflection band.

#### 3. CALCULATIONS AND EXPERIMENTAL RESULTS

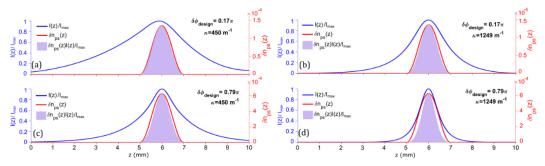


Figure 1. Refractive-index profile  $\delta n_{ps}(z)$  (red curve), normalized light intensity  $I(z)/I_{max}$  (blue curve) calculated at  $\lambda_{res}$ , and overlap product  $\delta n_{ps}(z)I(z)/I_{max}$  (purple area) versus position z for grating coupling coefficients of  $\kappa = 450 \text{ m}^{-1}$  and  $1249 \text{ m}^{-1}$  and designed phase shifts of  $\delta \phi_{design} = 0.17 \pi$  and  $0.79 \pi$ . The grating length is 10 mm and the phase-shift center is positioned at  $z_{ps} = 6 \text{ mm}$ . (Figure taken from Ref. [9].)

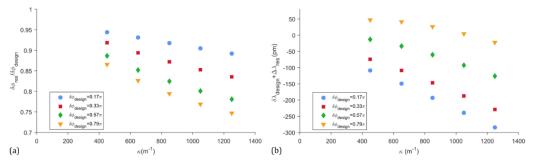


Figure 2. Simulation of (a) relative accumulated distributed phase shift  $\delta\phi_{res}$  /  $\delta\phi_{design}$  and (b) difference  $\delta\lambda_{res} = \delta\lambda_{design} + \Delta\lambda_{res}$  between resonance wavelength  $\lambda_{res}$  and Bragg wavelength  $\lambda_B$  versus  $\kappa$  for designed distributed phase shifts of  $\delta\phi_{design} = 0.17 \, \pi$ , 0.33  $\pi$ , 0.57  $\pi$ , and 0.79  $\pi$ . The grating length is 10 mm and the phase-shift center is positioned at  $z_{ps} = 6$  mm. (Figure taken from Ref. [9].)

We utilize the characteristic-matrix approach [11, 7]. The distribution of effective-refractive-index profile  $\delta n_{ps}(z)$  of equation (3), the simulated normalized light intensity  $I(z)/I_{max}$ , and the overlap product  $\delta n_{ps}(z)I(z)/I_{max}$ 

from equation (7) are displayed in Fig. 1. Two effects lead to an incomplete accumulation of designed phase shift. Firstly, when comparing Figs. 1(a) and 1(b) or Figs. 1(c) and 1(d), with increasing  $\kappa$  the light intensity is more confined to the phase-shift center, the light intensity accumulates less of the designed phase shift, hence  $\delta\phi_{res}$  decreases compared to the value of  $\delta\phi_{design}$  of equation (2) [Fig. 2(a)]. Consequently, in the case of a distributed phase shift, the resonance exhibits a negative wavelength shift  $\Delta\lambda_{res}$  compared to the designed  $\delta\lambda_{design}$  [Fig. 2(b)].

Secondly, when comparing Figs. 1(a) and 1(c) or Figs. 1(b) and 1(d), with increasing  $\delta\phi_{design}$  the light intensity is more confined to the region near the phase-shift center. The slow increase in effective refractive index between the two phase-shift ends and its center leads to an additional distributed reflection and tighter light confinement. Light propagating from the phase-shift center in both directions into the grating is more reflected for larger  $\delta\phi_{design}$ . For increasing  $\delta\phi_{design}$ , obtained by increasing  $\delta w_{ps}(z_{ps})$  and  $\delta n_{ps}(z_{ps})$ , this additional light confinement increases, hence the fraction  $\delta\phi_{res}/\delta\phi_{design}$  decreases [Fig. 2(a)], resulting in larger deviations  $\Delta\phi_{res}$  and  $\Delta\lambda_{res}$  [Fig. 2(b)].

We investigated experimentally amorphous  $Al_2O_3$  rib waveguides deposited by RF reactive sputtering onto a thermally oxidized silicon wafer [12] and micro-structured by reactive ion etching [13]. A corrugated homogeneous Bragg grating was inscribed into the  $SiO_2$  top cladding [2, 4]. The effective-refractive-index increase  $\delta n_{ps}(z)$  was achieved by tapering the waveguide structure according to the  $\sin^2$  function of Eq. (3) over a length of  $\ell_{ps}=2$  mm. The maximum increase  $\delta w_{ps}(z_{ps})$  assumed the values of (a) 0.1  $\mu$ m, (b) 0.2  $\mu$ m, (c) 0.35  $\mu$ m, and (d) 0.5  $\mu$ m. The corresponding values of the effective refractive index were calculated using a mode-solver software. According to Eqs. (2)–(5), it results in a designed phase shift  $\delta \phi_{design}$  of (a) 0.17  $\pi$ , (b) 0.33  $\pi$ , (c) 0.57  $\pi$ , and (d) 0.79  $\pi$ . The transmission spectra of these four waveguides were recorded. The observed resonances were systematically shifted to shorter wavelengths. Compared to the resonance wavelengths calculated from the overlap integral of Eq. (7), the experimental data show excellent agreement for the nominal  $\kappa$  of 650 m<sup>-1</sup>, when  $\delta \phi_{design}$  is small, indicating that the incomplete accumulation of designed phase shift  $\delta \phi_{design}$  strongly contributes to the observed wavelength deviation. A deviation at larger  $\delta \phi_{design}$  indicates that also fabrication errors play a role.

## 4. CONCLUSIONS

We have shown the fundamental nature of the wavelength deviation encountered in distributed-phase-shift DFB resonators. When the light intensity extends over a length comparable to the distributed-phase-shift region, the wavelength deviation is largely the result of an incomplete accumulation of the designed distributed phase shift by the light intensity that decays into the Bragg grating on both sides of the phase-shift center.

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