Large Q factor with small ring cavities

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ABSTRACT

Radiation losses are a critical problem of small-radius optical whispering gallery mode (WGM) resonators. This phenomenon is a fundamental barrier that prevents one to reduce the size of the cavity and increase the local electric field. We propose a structure around the cavity that can be built with existing technologies and that can suppress the radiation. We demonstrate this fact by a nearly exact formula that gives the enhancement of the Q factor in 2D and confirm our results by a series of numerical simulations in 3D. By decreasing mode volume while preserving photon lifetime, the performance of WGM cavities can be boosted and new limits can be reached in the many areas of research and applications where they are used, from cavity quantum electrodynamics to label-free biosensing.

Keywords: Whispering Gallery Modes, Bending losses, micro-ring resonators, small radius, Q factor.

1 INTRODUCTION

Circular dielectric cavities which support whispering gallery modes (WGM) are key elements of photonic integrated circuits [1]. Thanks to their combination of large Q factor and small volume V, photons carrying a locally large electric field can circulate for a long time without dissipation. As a result, all the forms of interaction between light and matter can be drastically enhanced: laser action [2], second- and third-harmonic generation [3], [4], scattering by particles [5], etc. In biosensing, the ratio Q/V is well known to determine the limit of detection (LOD) of the device [6].

On photonic integrated platforms, WGM cavities with Q factors as high as 10^6 can now be obtained with state-of-the art fabrication techniques [7], [8], [9]. However, in all the cases reported, such high values of Q are only reported for cavities with a large radius R compared to the optical wavelength λ, typically between 50μm and 1 mm. This necessity results from bending losses, which increase drastically as the R decreases below a threshold of about 10λ. Bending losses are the radiation of electromagnetic energy by a waveguide upon changes of direction of propagation; they represent a fundamental barrier to the reduction of microring radii as well as the radius curvature of bends in photonic circuits. Regarding bent waveguides, several attempts have already been made to reduce bending losses, notably by enclosing the waveguide by a second one [10], [11], [12]. However, the reduction of losses so obtained remains quite modest and, in the case of [10], mostly rests on an increased effective radius of curvature for the light path. In this communication, we report a way to control WGM bending losses that can lead to orders of magnitude enhancement of the Q factor. The proposed technique could allow one to maintain the large Q factors allowed by state-of-the-art material fabrication technique while diminishing the cavity radius R by several orders of magnitude. This could lead, for instance, to lower LOD in biosensing, larger efficiency in nonlinear photon generation for quantum optics sources, and micro-lasers with lower lasing threshold.

The structure proposed consists of set of concentric rings placed outside the cavity, forming a kind of 'dielectric sarcophagus' for WGM as depicted in Fig. 1. The innermost of these rings is typically located in the near field of the WGM. As we demonstrate, the Q factor can be increased by several orders of magnitudes in this way. Adding more rings to the external structure allows one to further improve the radial part of Q factor. Here, it is important to note that the external shells are too thin to guide light and that the mode energy remains concentrated in the cavity.

2 THEORY

To determine the actual bending losses of a cavity or a portion of bent waveguide is a complicated theoretical problem [13], [14]. However, it turns out that a simple and nearly exact formula can be derived for the ratio Q/Q_0 of the quality factor in the presence of the external structure (Q) to that of the bare cavity (Q_0). Consider a cylindrical geometry made of concentric dielectric shells, as in Fig. 1. The shell closest to the centre is the cavity, whose radiation we wish to suppress.

In the region of space defined by $r_{j-1} \leq r \leq r_j$ and of refractive index n_j, the electromagnetic is governed by

$$\psi(r) = [a_j J_\nu(n_j kr) + b_j Y_\nu(n_j kr)]\, e^{i\nu\theta - ikct}, \tag{1}$$

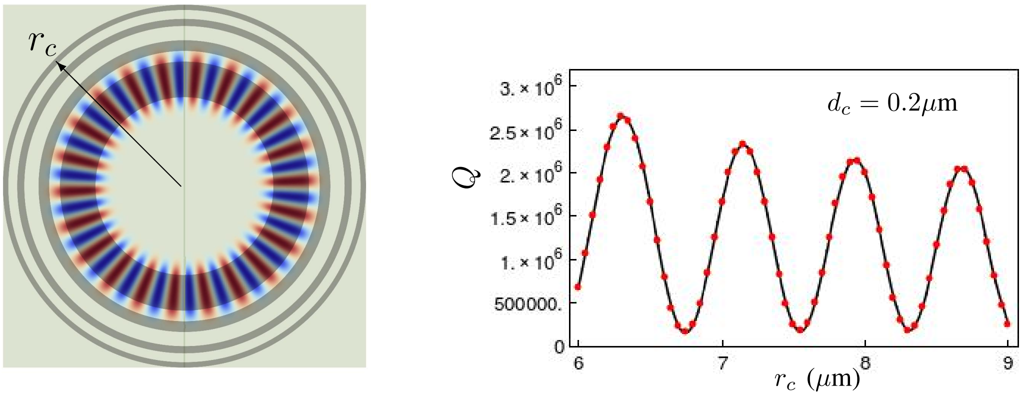

Figure 1. Left: sketch of a Al_2O_3 ring cavity with outer radius 3.2μm operating on a mode with orbital number $\nu = 22$ ($\lambda = 1.26\mu$m). Three external concentric rings are added to attenuate radiation. Without the external rings, $Q_0 \approx 15000$. Right: Q as a function of the radius of the outermost ring. The first and second rings have radii and thickness given by $(4.87, 0.23)\mu$m, and $(r_b, d_b) = (5.63, 0.21)\mu$m, respectively. The graph shows an enhancement $Q/Q_0 \approx 170$.

where $\psi = E_z$ for TE modes, $\psi = H_z$ for TM mode, r and θ are the usual cylindrical polar coordinates, c is the speed of light in vacuum and k is the complex wave number. The parameter ν is the azimuthal number; usually, for large R, $\nu \gg 1$, on the order of 1000 or more, which ensures that the radiation losses are very small. Here, we aim to achieve a large Q with small values of ν. In the central region, $(a_0, b_0) = (a_0, 0)$ in order to avoid divergence of the electromagnetic field. On the other hand, in the outermost region, Sommerfeld's radiation condition reads $(a_N, b_N) = (a_N, ia_N)$. The vectors $(a_0, 0)$ and (a_N, ia_N) can be related by a 2×2 scattering matrix S:

$$\begin{pmatrix} a_0 \\ 0 \end{pmatrix} = S \begin{pmatrix} a_N \\ ia_N \end{pmatrix}, \tag{2}$$

Above, $S = S^c S^s$, where S^c and S^s respectively describe the cavity and the dielectric shield. Their derivation follows straightforwardly from the continuity conditions between the various regions that constitute the whole system. The second of the two relations above, $S_{21}(k) + iS_{22}(k) = 0$ yields the characteristic equation for the whole system. Its solutions give the complex wave number $k = k_r - ik_i$ and the Q factor is given by $Q = k_r/(2k_i)$. By the same token, $S^c_{21}(k^c) + iS^c_{22}(k^c) = 0$ give the complex resonances of the bare cavity, associated to a quality factor $Q_0 = k^c_r/(2k^c_i)$.

To compute $k^c = k^c_r - ik^c_i$ for the bare cavity is a numerically easy task when ν is only moderately large. Conversely, a direct numerical resolution of the characteristic equation for the cavity+external rings rapidly becomes challenging as the number of rings increases. However, we have found that, with excellent numerical accuracy

$$\frac{Q}{Q_0} = \frac{k^c_i}{k_i} \approx \mathrm{Re}\left[\frac{S^s_{22}\left(k^c_r\right) - iS^s_{21}\left(k^c_r\right)}{S^s_{11}\left(k^c_r\right) + iS^s_{12}\left(k^c_r\right)}\right], \tag{3}$$

which only requires the knowledge of k^c_r associated to the bare cavity. This formula allows one to simply and rapidly evaluate the validity of a set of geometrical parameters for the outer shells. Moreover, it suggests to design the shielding structure by adding and optimising one shell at a time. Starting from a single shell, one can easily optimise its radius r_a and radial thickness thickness d_a. Next, a second shell with radius $r_b > r_a$ can be added and optimised and so on, until the entire structure is completed.

3 RESULTS

An illustration of the accuracy of formula (3) is given in Fig. 1, describing a mode with azimuthal number $\nu = 22$ in an Al_2O_3 cavity of external radius $R = 3.2\mu$m. We choose this material because it has been demonstrated to be an advantageous, CMOS compatible, host for rare-earth dopant and to hold great potential to integrate micro-lasers in photonic platforms [15], [8]. However, its index of refraction is only 1.65, a rather small value compared to other PIC materials. For the above parameters, $\lambda \approx 1.26\mu$m, which is in the emission range of Yb atoms, that can dope Al_2O_3 for lasing. In the absence of the external shielding structure, we compute that $Q_0 \approx 15000$. By optimizing a three-shell structure, one finds that the quality factor can be raised to $Q \approx 2.5 \times 10^6$ as far as radiation losses is concerned. Such value correspond to the state-of-the-art material limit of Q.

To check the validity of our design in 3D, we have carried out a series of 3D simulations. In, Fig. 2, we show the simulation of a ridge-waveguide Al_2O_3 on SiO_2. By placing a single outer ring at an optimal radius, we obtain an enhancement of quality factor $Q/Q_0 \approx 8$. Fig. 2 shows that the field stays confined in the cavity in the presence of the dielectric shield and illustrates how the intensity distribution is attenuated in the far field. Adding further rings, one can further improve Q (not shown).

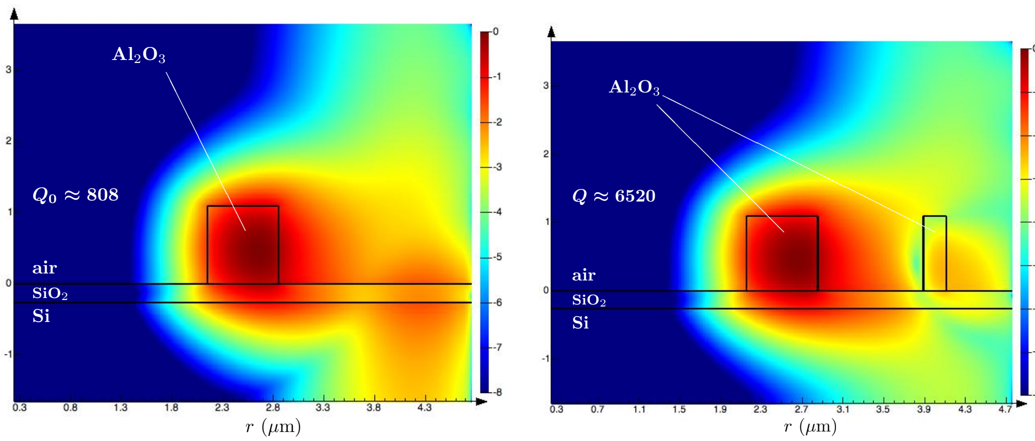

Figure 2. Mode distribution (logarithmic color scale) of a ridge Al_2O_3 ring cavity over a SiO_2 base. Bare cavity radii: inner radius $r_1 = 2.15\mu$m, $r_2 = 2.85\mu$m. Height: 1.1μm. SiO_2 thickness: 0.25μm, $\lambda = 1.26\mu$m.

4 CONCLUSION

In conclusion, we have demonstrated a way to drastically enhance the quality factor of microring cavities and bent waveguides. A simple theory has been derived that accurately predicts the gain that can be achieved by complementing the cavity with external rings. The proposed geometries have been demonstrated with Al_2O_3 but it is clear that it is not restricted to that particular material and that this structure can be made with silicon photonics or silicon nitride. Regarding the excitation of microring surrounded by other ring, one can resort to buried waveguide, as for instance in [8].

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