Analysis of the sensitivity of Mach-Zehnder interferometer filter to fabrication tolerances through elementary effect test

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Abstract: Sensitivity analysis method is applied to the Mach-Zehnder interferometer filter to assess how the fabrication processes variations of some geometrical parameters can influence the performance of the photonics devices.

1. Introduction: Manufacturing variations are becoming an unavoidable issue in modern fabrication processes, because can have a strong effect on the functionality of the fabricated circuits, limiting the sustainable complexity and hampering the possibility to achieve high yield of the fabrication processes. Therefore, it is crucial to be able to include stochastic uncertainties in the design phase [1]. Sensitivity analysis techniques [2] are hence of fundamental importance for the identification of the uncertain parameters (e.g. waveguide width, doping level, height of the layers) that have the greatest influence on the overall circuit performance. Typically, these techniques are adopted to screen the whole range of uncertain parameters, with the aim to isolate the subset which has the strongest effect on the outputs, thus reducing the number of parameters that should be more carefully investigated to maximize the number of fabricated devices fulfilling design specifications [3]. The most common sensitivity techniques proposed in literature include the elementary effect test (Morris method) and variance based sensitivity analysis (Sobol). In this paper we focus on Morris method to analyse the behaviour of complex photonic circuits subject to fabrication uncertainties. In particular, we exploit the method of Morris to evaluate the variability of the 3-dB bandwidth of a seventh order Mach-Zehnder interferometer filter exposed to random fluctuations of the waveguide width and the gap of coupling regions.

2. Theoretical background: Morris method [4], is based on the computation of differential metrics, called elementary effects, evaluated on a statistical basis on the whole space of variation of the input parameters of an assigned model. Being f(x) the model whose sensitivity needs to be assessed, for a given vector of uncertain parameters x belonging to sample space Ω, the elementary effect EE_i of the input factor x_i on f is defined as the incremental ratio:

$$EE_i(x) = \frac{f(x_1, x_2, \dots, x_i + \Delta x_i, \dots, x_k) - f(x_1, x_2, \dots, x_i, \dots, x_k)}{\Delta x_i}. \quad (1)$$

The statistical parameters (mean and variance) of elementary effects EE_i provides a direct measure of the influence of the factor x_i on the output function f. In particular, a high value for the mean indicates that the considered parameter has an important overall influence on the output function f.

3. Sensitivity analysis of a seventh-order Mach-Zehnder interferometer filter: The goal is to measure the impact of the random variation of the waveguide width and gap of coupling regions on the 3- dB bandwidth of a seventh-order MZI filter. The designed MZI filter has 3-dB bandwidth of $BW_0 = 27.5GHz$ and the corresponding coupling coefficients for the eight directional couplers are $K_1 = K_8 = 0.9914$, $K_2 = K_7 = 0.0004$, $K_3 = K_6 = 0.0690$ and $K_4 = K_5 = 0.3489$. This value provided an in-band isolation of 20 dB. All the seven stages have the same imbalance length of $134.3\mu m$ and the same effective index $n_{eff} = 2.23$ (SOI technology) with nominal waveguide width of $W_i = 407.7nm$. The nominal gap in eight coupling regions of coupler is $g_i = 0.3\mu m$. The free spectral range of the design is $100GHz$. Fig. 1a shows the transfer function of the filter based on Mach-Zehnder interferometer at bar and cross ports. In the described test case, random process variations are represented by noise applied to the relevant parameters of the filter, that is to the waveguides width and gap of the coupling regions of MZI filter. The vector $[\,\widetilde{W}_1\, \widetilde{W}_2\, \widetilde{W}_3\, \widetilde{W}_4\, \widetilde{W}_5\, \widetilde{W}_6\, \widetilde{W}_7\, \tilde{g}_1\, \tilde{g}_2\, \tilde{g}_3\, \tilde{g}_4\, \tilde{g}_5\, \tilde{g}_6\, \tilde{g}_7\, \tilde{g}_8]$ of the random parameters represents the process variation variables that are assumed to be Gaussian distributed with $\mu_{width} = 407.7nm$, $\mu_g = 0.3\mu m$, $\sigma_W = 30pm$ and $\sigma_g = 1nm$. The considered fabrication process variation can heavily affect the performance of the circuit, as shown in Fig 1a with thin grey lines. The 3-dB passband have large fluctuation (see Fig. 1b) even under relatively small process variations. Hence, our goal is to assess the dependence of 3–dB filter bandwidth on waveguides width and gap of coupling regions of coupler and to find the most influential random input parameter which is contributing to the output variability. The obtained results for Morris sensitivity analysis with $r \times (M + 1)$ (where r is number of finite differences and M is number uncertain input parameters) model evaluations (where $r = 1000$ and $M = 15$) are presented in Fig. 1c, which shows the mean μ^* versus the standard deviation of the elementary effects for 3-dB bandwidth of filter as a function of the random waveguides width and gap of coupling regions. Result obtained for analysis took around 4-5 minutes (16000 samples) with computational speed of core i7 (2.4 GHz) with 16 GB ram. The circles in the Fig. 1c shows the impact of each waveguide width and the gaps of coupling region in defining bandwidth of filter while squares are the 95% confidence bounds of the corresponding circles. The more a point is placed on the right of the horizontal axis, the more influential is the factor. The higher up a point is along the vertical axis, the larger is its degree of interactions with other factors. By observing Fig. 1(c), it is possible to conclude that for defining the bandwidth of filter, the waveguide width is more important than the gap of coupling region for a given variations of 1 nm for the gaps and 30 pm for waveguide width.

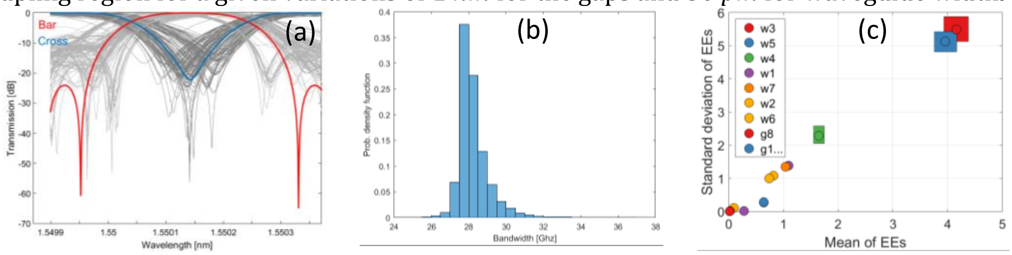

Figure 1:(a) Transfer functions of the bar and cross port of nominal design when fabrication variations is applied on the waveguide width and gap of coupling regions (grey thin lines). (b) Probability density function of the 3-dB bandwidth of the filter. Nominal bandwidth is 27.5 GHz. (c) Result of the Morris sensitivity analysis performed on the 3-dB bandwidth of the filter. Horizontal axis shows absolute mean (μ^*) of elementary effects. Vertical axis shows the standard deviation of elementary effects.

4. Conclusion: Sensitivity of the filter bandwidth has been analysed and the most influential random input parameter (waveguide width) have been identified. The discussed techniques can be easily extended to take into account also the influence of other parameters on the different responses of the investigated devices.

5. References

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