

# Threshold Analysis of 2-D Gain and Index Coupled Photonic Crystal Lasers

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In this work we compare threshold mode operation of Index and Gain coupled photonic crystal lasers. The active region of the laser structure consists of two materials - background material with high refractive index and cylindrical rods/holes arranged in square or triangular lattice with low refractive index. It is worth noting that some other rods geometries might be considered in future works, e.g., [1]. For the two structures and two lattices types two orthogonal polarizations, i.e., transverse magnetic and transverse electric, are investigated. From the eight possible cases the most favorable design in terms of lasing threshold is determined. The analysis is based on the coupled mode theory, which gives sets of coupled equations for each of the investigated cases. An exemplary set of equations for square lattice index and gain coupled structure with TM polarization is given by [2,3]:

$$-\frac{\partial}{\partial x} E_1^s + (\alpha - i\delta) E_1^s = (i\eta_3 - \alpha_3) E_3^s + (i\eta_2 - \alpha_2)(E_2^s + E_4^s), \quad (1)$$

$$\frac{\partial}{\partial x} E_3^s + (\alpha - i\delta) E_3^s = (i\eta_3 - \alpha_3) E_1^s + (i\eta_2 - \alpha_2)(E_2^s + E_4^s), \quad (2)$$

$$-\frac{\partial}{\partial y} E_2^s + (\alpha - i\delta) E_2^s = (i\eta_3 - \alpha_3) E_4^s + (i\eta_2 - \alpha_2)(E_1^s + E_3^s), \quad (3)$$

$$\frac{\partial}{\partial y} E_4^s + (\alpha - i\delta) E_4^s = (i\eta_3 - \alpha_3) E_2^s + (i\eta_2 - \alpha_2)(E_1^s + E_3^s), \quad (4)$$

In the above equations  $E_i^s$ ,  $i = 1,2,3,4$  are the four basic electric field amplitudes propagating in four directions of the photonic crystal cavity, i.e.,  $+x$ ,  $-x$ ,  $+y$ ,  $-y$ . The coupling in the structure is expressed by  $\eta_2$  and  $\eta_3$ . The periodic gain is stated by  $\alpha_2$ ,  $\alpha_3$  - which are the higher order gain components, which for Index coupled structure are assumed to be 0, whereas for Index and Gain Coupled structure are assumed nonzero. The gain coefficient  $\alpha$  describes a threshold gain in the lasing medium. The coupling coefficients are responsible for orthogonal (e.g., the coupling of  $E_1^s$  to  $E_2^s$  and  $E_4^s$ ) and backward (e.g., the coupling of  $E_1^s$  to  $E_3^s$ ) coupling. In equations (1) - (4) the Bragg frequency deviation is given by the following expression:

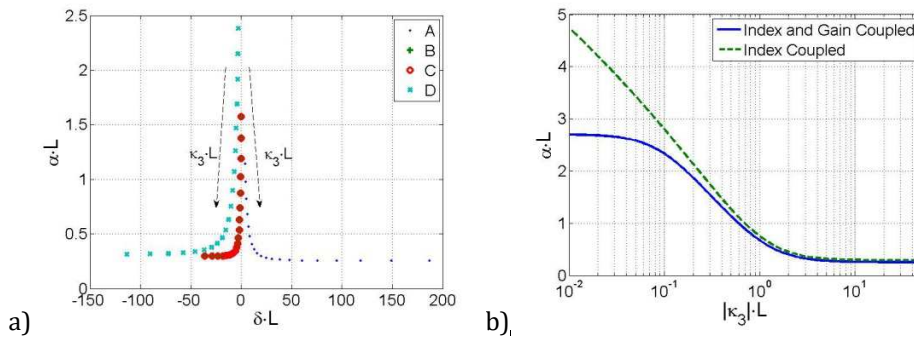
$$\delta = (\beta^2 - \beta_0^2)/2\beta \approx \beta - \beta_0^s. \quad (5)$$

Equations (1) - (4) are solved using the finite difference method with appropriate boundary conditions. Obtained solutions describe each mode by its threshold distributions  $E_i^s$ , threshold gain, and Bragg frequency deviation.

The calculations are conducted for square and triangular lattices assuming that both have the same filling factor (the fraction of rods area to the structure area)  $f = 0.16$ . Here, for the presented case of square lattice it is assumed that  $|\eta_2 L| = 4.1$ , and

$|\eta_3 L| = 2$ , where  $L$  denotes cavity length.

An example solution of equations (1) – (4) showing the behavior of threshold gain in terms of Bragg frequency deviation and coupling coefficient are given in figure 1. Figures 1a shows the comparison of threshold gain for four fundamental modes in case of Index and Gain Coupled structure. Figure 1b depicts (for the mode A) the development and comparison of the threshold gains in a wide range of the coupling coefficient for the Index and Index and Gain Coupled structures.



**Fig. 48. Square lattice structure with TM polarization. The dependence of normalized threshold gain versus a) Bragg frequency deviation; and b) normalized coupling coefficient for mode A for Index and Gain Coupled (solid line) and Index Coupled (dashed line) structures [3].**

The outcome shows that the nonuniformity of the gain in the low index contrast structures has a strong impact on the threshold gain, by lowering it. Consequently, by inducing gain coupling in the Index Coupled structure, it is possible to lower threshold gain particularly for low index contrast photonic crystals. The overall analysis shows that the lowest threshold is achieved for the Index and Gain Coupled structures, and it takes lower values for higher refractive index contrast in case of transverse magnetic polarization. This outcome helps understand the principles of PC band-edge laser operation and it may be useful in supporting the design process of PC laser structures.

## References

- [1] Y. Liang, C. Peng, K. Sakai, S. Iwahashi, and S. Noda, *Three dimensional coupled-wave analysis for square-lattice photonic crystal surface emitting lasers with transverse-electric polarization: Finite-size effects*, Optics Express, vol. 20, no. 14, pp. 15945-15951, 2012
- [2] M. Koba, P. Szczepanski, T. Osuch, *Threshold Mode Analysis of 2-D Square and Triangular Lattice Gain and Index Coupled Photonic Crystal Lasers*, IEEE Journal of Quantum Electronics, vol. 50, no. 7, pp. 554-562, 2014
- [3] M. Koba, *Advances in Photonic Crystals*, InTech, Rijeka, 2013
- [4] M. Koba, P. Szczepanski, *Photonic Crystals - Introduction, Applications and Theory*, InTech, Rijeka, 2012