Amplitude and phase error correction in $3 \times 3$ MMIs

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Abstract: We present a correction algorithm that is able to reduce the influence of amplitude and phase errors in a $3 \times 3$ MMI. When used in a phase estimator, our method reduces the maximum estimation error from $8^\circ$ to $0.3^\circ$.

1 Introduction

A wavelength meter can be used for various applications ranging from monitoring laser diodes to Fiber Bragg Grating read-out systems. On-chip wavelength estimation can be realized by using an imbalanced Mach-Zehnder Interferometer combined with an optical hybrid. In such a system, the MZI is used to map a shift in wavelength to a phase difference between the arms. The phase difference between arms is then estimated by the optical hybrid. A $3 \times 3$ Multimode Interference coupler (MMI) can be used as a 120° optical hybrid. By using the outer two inputs of the MMI, a high level of symmetry is obtained, resulting in optimum balance. Figure 1 shows a schematic of the readout system. Ideally, the outputs of the MMI are perfectly balanced and spaced by 120 degrees in the phase domain. In practice, there will be imbalances of a few tenths of a dB and a phase error of a few degrees. This results in a wrong estimation of the phase difference between the MZI arms. To improve the accuracy of the phase estimation, a correction algorithm can be applied. Reyes et al. proposed a linear correction scheme, but the system is also subject to non-linear distortion. In this paper we describe a calibration method that also corrects for non-linear distortion, thereby reducing the estimation error caused by imperfections in the MMI.

![Fig. 1: Schematic of a wavelength meter system. SSC: spot size converter, PD: photo-diode.](image)

2 System

In the system shown in figure 1, there are three photo-diodes connected to each of the MMI outputs. The purpose of this system is to determine the phase difference $\Delta \phi$ between the two MZI arms from the measured photo-currents. The current $I_i$ that can be measured at detector $PD_i$ is described by the following set of equations.

$$I_1 = 0.5 \cdot R \eta P_{in} \left( |a|^2 + |ac|^2 + 2|ac\alpha| \cos(\Delta \phi - \phi_{\text{MMI}}) \right)$$

$$I_2 = 0.5 \cdot R \eta P_{in} \left( |b|^2 + |ab|^2 + 2|ab\alpha| \cos(\Delta \phi) \right)$$

$$I_3 = 0.5 \cdot R \eta P_{in} \left( |c|^2 + |ca|^2 + 2|ca\alpha| \cos(\Delta \phi + \phi_{\text{MMI}}) \right)$$

with $\Delta \phi = 2\pi N e f f (\lambda) \Delta L \lambda^{-1}$, $\phi_{\text{MMI}} = \arg\{c\} - \arg\{a\} = 120^\circ + \epsilon_\phi$, and with $R$ the detector responsivity, $\eta$ the input coupling efficiency comprised of SSC loss and $1 \times 2$ splitter loss, $P_{in}$ the applied input power, $\alpha$ the MZI balance, $\epsilon_\phi$ the phase deviation from 120°.

3 Calibration method

Our calibration method consists out of three steps. First, the device has to be characterized by performing a wavelength sweep over a single free spectral range of the MZI, while recording the photo-currents of the detectors. These measurements are subject to noise, among which the dark-current. In the absence of any input light, the mean of the dark-current of every detector can be measured, and then subtracted from the photo-current before further calculations. The second step is to fit the measured results to the equations of (1). Using a non-linear least squares fit method, the coefficients $a$, $b$, $c$, $\alpha$, $\phi_{\text{MMI}}$ and of the combined term $0.5 \cdot R \eta P_{in}$ can be estimated. Additionally, the phase difference $\Delta \phi$ needs to be expressed in terms of $\lambda$. This results in three more fitting parameters: $C_1, C_2, C_3$.

$$\Delta \phi = 2\pi \lambda^{-1} C_1 + (\lambda - \lambda_\phi) C_2 + (\lambda - \lambda_0)^2 C_3$$

Finally, the third step is using the obtained parameters to correct for a non-ideal MMI response. In order for the correction scheme to work, the applied input power needs to be known. An estimate of this value can be obtained by summing the three photo-currents.

$$I_1 + I_2 + I_3 = 0.5 \cdot R \eta P_{in} \left[ (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2) + 2|\alpha(2|ac| \cos(\phi_{\text{MMI}}) + |b|^2) \cos(\Delta \phi) \right]$$

(3)
Using \( \cos(\phi_{\text{MMI}}) = \cos(120^\circ + \epsilon_0) \approx -\frac{1}{2} - \frac{\sqrt{3}}{2} \epsilon_0 \), results in
\[
I_1 + I_2 + I_3 \approx 0.5 \cdot R \eta P_{\text{in}} \left[ (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2) + 2|\alpha|(|b|^2 - |ac| - \sqrt{3}|ac|\epsilon_0) \cos(\Delta \phi) \right]
\]
When phase and amplitude errors are small, the term \( (|b|^2 - |ac| - \sqrt{3}|ac|\epsilon_0) \) can be neglected.
\[
I_1 + I_2 + I_3 \approx 0.5 \cdot R \eta P_{\text{in}} (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2)
\]
This means all measurements can be normalized by dividing the photo-currents by the sum of the photo-currents and then multiplying by \( (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2) \). It can now be shown that the following relations hold:
\[
\cos(\Delta \phi) \approx \frac{X (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2) + (1 + |\alpha|^2)(|a|^2 - 2|b|^2 + |c|^2)}{4|\alpha||b|^2 - 4|\alpha|ac}\cos(\phi_{\text{MMI}})
\]
\[
\sin(\Delta \phi) \approx \frac{Y (1 + |\alpha|^2)(|a|^2 + |b|^2 + |c|^2) - \sqrt{3}(|\alpha|^2 - 1)^2(|c|^2 - |a|^2)}{4\sqrt{3}|ac|\sin(\phi_{\text{MMI}})}
\]
where
\[
X = (2I_2 - I_1 - I_3)(I_1 + I_2 + I_3)^{-1}
\]
\[
Y = \sqrt{3}(I_1 - I_3)(I_1 + I_2 + I_3)^{-1}
\]
The phase difference \( \Delta \phi \) can now be easily obtained by calculating the arctangent of the sine and cosine terms in (5) and (6).
\[
\Delta \phi = \arctan \left( \frac{\sin(\Delta \phi)}{\cos(\Delta \phi)} \right)
\]

4 Performance and conclusion
To test the performance of the proposed calibration method, we simulated the estimation error as a function of the MMI phase error and the imbalance. The value for \( \alpha \) was kept constant at 0.8, while the values for \( a \) and \( b \) were kept constant at \( \sqrt{0.3} \) and \( \sqrt{0.314} \) respectively. The value of \( c \) changed with the imbalance of the MMI. The resulting estimation errors without correction and with correction are shown below. Due to the fixed imbalance between \( a \) and \( b \), the minimum estimation error does not occur at \( (0,0) \). The simulation range corresponds to a typical width error of \( \pm 200 \) nm in a 12 \( \mu \)m wide indium phosphide based MMI. Our method is able to reduce the estimation error from \( 8^\circ \) to less than \( 0.3^\circ \) for a wide range of MMI errors. This allows for a more accurate measurement of the wavelength of incoming light.

![Fig. 2: Estimation error without (a) and with (b) correction algorithm. Please note the difference in scale.](image)

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References