An enhanced wavelength tolerant design for the Generation of N-partite single photon W-states

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Abstract: Recently integrated optic solutions for generation of N-partite W-state have been proposed using a 1-D array of 'N' identical single mode optical waveguides. The proposed design exhibits very low tolerance to the input wavelength. Here we propose an alternative design of a waveguide array consisting of (2N-1) waveguides to create 'N' partite W state that exhibits much greater tolerance to the input wavelength thus making it easy for experimental realization.

It is well known that entanglement plays a crucial role in various phenomena in the field of quantum computation[1], quantum cryptography[2-3], teleportation[4], quantum key distribution etc. Among various kinds of non-classical entangled states, W states are important as their entanglement is less fragile against loss of any qubit whereas the other class of entangled states i.e. GHZ states become fully separable after qubit loss[5]. For the realization of such non-classical W states, quantum optics provides an elegant platform[6].

Recently integrated optic solutions for generation of W-states have been proposed using a 1-D array of N identical single mode optical waveguides, which are coupled via nearest neighbor interaction[7]. The flow of a single photon through such structure is governed by a set of Heisenberg equation of motion:

$$i\frac{d\hat{a}^\dagger}{dz} = K\hat{a}^\dagger \qquad (1)$$

where $\hat{a}^\dagger$ is the creation operator representing the z dependence of the field in each of the waveguides and given by $\hat{a}^\dagger = \left[\hat{a}_1^\dagger, \hat{a}_2^\dagger, ...\hat{a}_N^\dagger\right]^T$ and K is the coupling matrix representing the coupling co-efficient between adjacent waveguides. Solution of eqn. (1) gives the evolution of the z-dependence of the field as

$$\hat{a}^\dagger(z) = e^{-iKz}\hat{a}^\dagger(0) \qquad (2)$$

Here the term e^{-iKz} acts as the z-evolution operator corresponding to the z-dependence of field in each waveguide. It is well known that for an array consisting of three equally spaced identical waveguides, for a single photon incident in the central waveguide a three partite W-state is formed at a propagating distance $z_0 = \left[\tan^{-1}\left(\pm\sqrt{2}\right) + n\pi\right] \Big/ \sqrt{2}\kappa_0$. The major problem with the above mentioned design is that the tolerance to the wavelength of the input single photon for the generation of the W-state is very critical. Figure 1 shows the variation of probability of finding the photon in an array of 3 equally spaced ($d = 12\mu m$) identical waveguides as a function of input wavelength, for a device of length $z = 2.48\,cm$ designed to provide a W-state at $\lambda_0 = 1\mu m$. Even a change of the input wavelength by ± 0.5 nm. (≈ ±0.05 %) leads to deviation from the W state with variation in probabilities of about ± 0.5 %. Such critical tolerance can lead to problems in practical applications of such devices.

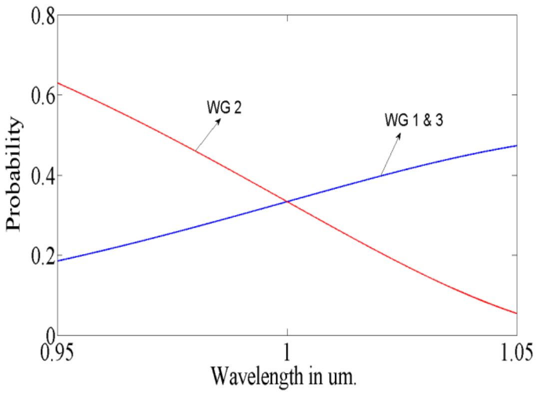

Fig. 1: Wavelength variation of probability of finding a single photon in an array of 3 equally spaced waveguides

To overcome this problem and for a better experimental feasibility of *W* states, we propose an alternative design of a waveguide array consisting of $(2N-1)$ waveguides to create an *N* partite *W* state that exhibits much greater tolerance to the input wavelength.

For example to generate a 3-partite *W*-state, we have considered an array of five waveguides with unequal coupling coefficients given by $\kappa_{2,3} = \kappa_{3,4} = \kappa_0$ and $\kappa_{1,2} = \kappa_{4,5} = \tilde{\kappa}_0$. Now, if a single photon is incident at the central waveguide, then the initial state of the photon at $z = 0$ i.e. $|\psi_{in}(z=0)\rangle = |00100\rangle = \hat{a}_3^\dagger(0)|00000\rangle_{z=0}$ evolves along the propagating distance to a superposition of states

$$|\psi_{out}(z)\rangle = \sum_{p=1}^{5} (T)^*_{p,3}\, \hat{a}_p^\dagger(z)|00000\rangle_z \qquad (4)$$

where

$$(T)^*_{1,3}(z) = (T)^*_{5,3}(z) = \frac{\kappa_0\tilde{\kappa}_0}{2\kappa_0^2 + \tilde{\kappa}_0^2}\left[-1 + \cos\left(\sqrt{2\kappa_0^2 + \tilde{\kappa}_0^2}\, z\right)\right] \quad (5)$$

$$(T)^*_{2,3}(z) = (T)^*_{4,3}(z) = -\frac{i\kappa_0}{\sqrt{2\kappa_0^2 + \tilde{\kappa}_0^2}}\left[\sin\left(\sqrt{2\kappa_0^2 + \tilde{\kappa}_0^2}\, z\right)\right] \quad (6)$$

$$(T)^*_{3,3}(z) = \frac{\tilde{\kappa}_0^2}{2\kappa_0^2 + \tilde{\kappa}_0^2} + \frac{2\kappa_0^2}{2\kappa_0^2 + \tilde{\kappa}_0^2}\left[\cos\left(\sqrt{2\kappa_0^2 + \tilde{\kappa}_0^2}\, z\right)\right] \quad (7)$$

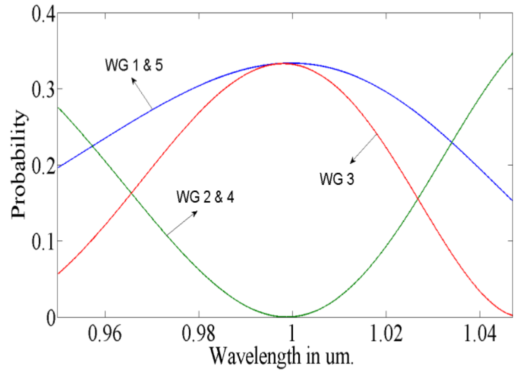

Fig. 2: Wavelength variation of probability of finding a single photon in an array of 5 waveguides (according to our design)

Now, our aim is to generate a 3 partite *W* state in the array of 5 waveguides i.e probability of finding the photon in WG#1, WG#3, WG#5 should be the same (equal to 1/3) and in WG#2, WG#4 it should be zero. This can be achieved by a proper selection of ratio of the propagation distance (z) to the coupling coefficients $(\kappa_0, \tilde{\kappa}_0)$. To demonstrate the idea, simulations have been performed for an array of length $z = 7.24\,cm$, consisting of planar waveguides each of width 4 µm. Figure 2 shows the variation of probability of finding the photon in an array of 5 waveguides as a function of input wavelength. The figure clearly demonstrates the much enhanced tolerance of ± 2.5 nm. (≈ 0.25%) to the input wavelength thus clearly showing improved performance of the proposed configuration. Simulations for the generation of 5 partite *W*-states using 9 waveguides show similar enhanced tolerance. The enhanced spectral bandwidth of the proposed device can be used for the generation of *W*-states using pulsed single photon sources with picosecond pulse width.

In conclusion, we have shown that an appropriately designed 1-D array of $(2N-1)$ identical waveguides can lead to *N*-partite *W*-state with much greater tolerance to the input wavelength than the earlier proposed designs.

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