Optical devices for the Discrete Fourier Tranform

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***Abstract*—We review the design guidelines for devices that implements the DFT in the optical domain. We also introduce a novel planar architecture to perform the DFrFT, based on an AWG configuration.**

Fourier transform; waveguide grating router; Mach Zehnder interferometer; orthogonal frequency division multiplexing

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is the key technology for a large variety of high-speed digital communications over bandwidth-constrained physical media, both wireless and copper wired. The basic idea of multicarrier modulation dates back to 1960s, but it became a practical reality only when the appearance of mass market applications coincided with the availability of efficient electronic digital signal processing (DSP). The two key features for the widespread of OFDM technology are the orthogonality of the subacarriers that enhances the spectral efficiency, and the electronic signal processing to equalize the channel distortion.

OFDM is gaining larger and larger research interest also among the fiber optics community, and at the moment is one of main R&D topic. The modulation/detection combinations for optical OFDM include optical intensity modulation (IM) with direct detection (DD), optical modulation with coherent detection, and all-optical modulation with coherent detection [1]. In the latter case, the discrete Fourier transform (DFT) and the inverse discrete Fourier transform (IDFT) are (analogically) implemented in the optical domain, avoiding high-speed and power consuming DSP, analog to digital converter (ADC) and digital to analog converter (DAC). Recently, the record transmission of a 26 Tb/s transmission has been demonstrated [2].

The implementation of a device that performs the DFT in the optical domain may have different physical implementations, and in the present paper some optical architectures are analyzed. A novel configuration to perform the discrete fractional Fourier transform (DFrFT) is also described, that can be advantageously used in an optical OFDM system.

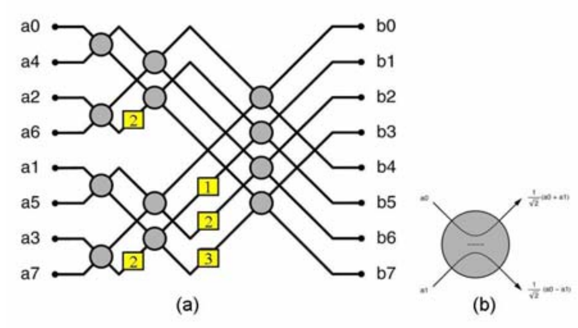

Figure 1. From [4]: (a) meshed fiber network composed of asymmetric couplers and phase shifters that performs the DFT of a parallel input. The phase shifters are represented by the boxes of value -q2π/N (q is the number inside the box). (b) asymmetric coupler.

II. OPTICAL DFT DEVICES

The first architecture to optically implement the DFT was first proposed by M. E. Marhic [3] and later restated by A. E. Siegman [4]. The meshed fiber network of Fig. 1(a) is composed of asymmetric couplers and phase shifters to perform the DFT of a parallel input signal

$$b_m = \sum_{n=0}^{N-1} a_n \exp\left(-j\frac{2\pi mn}{N}\right) \quad \left(\mathrm{m} = 0,1,2,..,\mathrm{N}-1\right), \cdot \qquad (1)$$

According to the Cooley-Tukey algorithm, the N-order DFT of a parallel input signal is evaluated by a chain of log_2N stages, each stage is composed of an asymmetric *2X2* *3*dB coupler (Fig. 1(b)), that implements the second-order DFT. However, to be used in optical communications, where data are serially transmitted over a fiber, a serial to parallel (S/P) converter is required. To overcome this limitation, a restatement of the optical DFT approach was formulated for a discrete serial input [5]: the Nth-order DFT of a sequence of optical bits is obtained by a chain of log_2N stages, each of them composed of a Mach Zehnder Interferometer (MZI) with an output asymmetric coupler, that corresponds to the second-order DFT, as shown in Fig. 2.

A different architecture for the optical Nth-order DFT has been also proposed with the arrayed waveguide grating (AWG) configuration shown in Fig. 3.

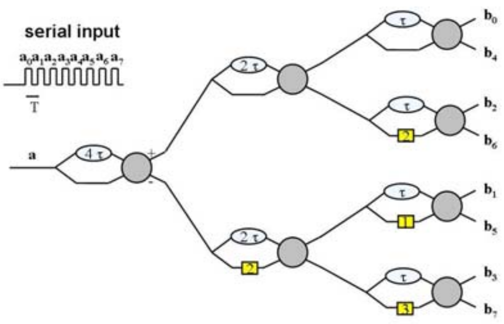

Figure 2. From [6]: tree architecture composed of MZIs with asymmetric couplers and phase shifters that performs the DFT of a serial input. The phase shifters are represented by the boxes of value -q2π/N (q is the number inside the box).

In this case, the array pitches d and d_o have to satisfy the condition

$$d = d_o = \sqrt{\frac{\lambda R}{N}} \qquad l = R \cdot \tag{2}$$

where l and R are the slab length and radius of curvature, respectively. In addition, the delay $\Delta\tau$ introduced by two adjacent waveguides should equate the inverse of the input bit rate.

III. OPTICAL DFRFT DEVICE

It is important to observe that the second slab coupler (Fig. 4(a)) of the AWG performs the spatial DFT of a parallel input signal, and it is equivalent to the two-lens system shown in Fig. 4(b). It is well known that the light distribution in the output plane is the Fourier transform of the input object, and this concept is at the basis of the whole optical information processing. By equally spacing the input and output waveguides, that satisfy Eq. (2), we obtain the DFT. The waveguide grating with delay $\Delta\tau$ is used only to perform the S/P conversion.

If we change the slab length and the curvature radius so that they satisfy the condition

$$d = d_o = \sqrt{\frac{\lambda R}{N}} \qquad R = R_1 \cot\left(p\frac{\pi}{4}\right) \qquad l = R_1 \sin\left(p\frac{\pi}{2}\right) \tag{3}$$

where R_1 is a constant length, we obtain the DFrFT of order $0<p<2$.

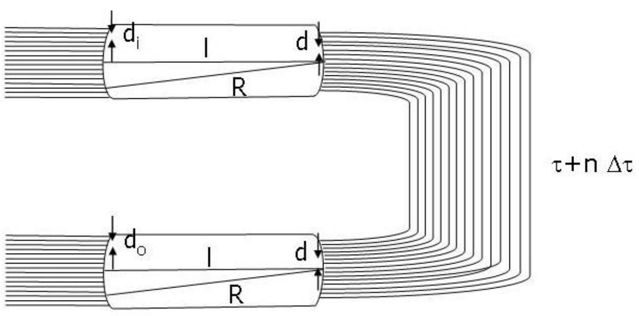

Figure 3. AWG architecture that performs the DFT.

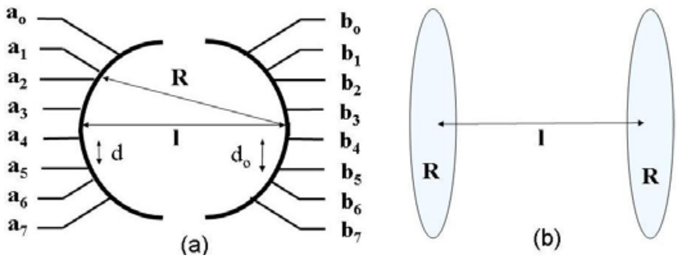

Figure 4. (a) slab coupler configuration. (b) two lenses configuration.

The analog fractional Fourier transform of order p of a signal $x(t)$ is defined as

$$\tilde{x}(u) = F^p\left\{x(t)\right\}(u) = \left|\sin\left(p\frac{\pi}{2}\right)\right|^{-\frac{1}{2}} e^{j\frac{\pi}{4}\left\{p - sign\left[\sin\left(p\frac{\pi}{2}\right)\right]\right\}} \cdot \int_{-\infty}^{\infty} x(t) e^{j\pi\left[\left(t^2+u^2\right)\cot\left(p\frac{\pi}{2}\right) - 2tu\csc\left(p\frac{\pi}{2}\right)\right]} dt \tag{4}$$

and it is a generalization of the standard Fourier transform, and satisfies the additivity property

$$F^p\left\{F^q\left\{x(t)\right\}\right\} = F^{p+q}\left\{x(t)\right\} \; . \tag{5}$$

OFDM schemes based on the DFrFT can be advantageously used in high-speed fiber systems, because the corresponding subcarriers satisfy the orthogonality condition; in addition they present a chirped behavior that can be used to compensate chromatic dispersion. Furthermore, the peak-to-average power ratio (PAPR) corresponding for OFDM modulation can be largely reduced if the DFrFT is used.

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