Integrated Optics Implementation of Universal Quantum Gates, Bell States Preparation Circuit, Quantum Relay and Quantum LDPC Decoders

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Abstract— **We show that arbitrary family of universal quantum gates can be implemented in integrated optics based on single optical hybrid/Mach-Zehnder interferometer/directional coupler and highly nonlinear optical fibers (HNLFs). We also show how to implement the Bell states preparation circuit and quantum relay, needed in quantum teleportation systems, using the same technology. Finally, we study the implementation of sparse-graph quantum decoders in integrated optics.**

Keywords- quantum teleportation; quantum information processing; integrated optics devices; quantum error-correction codes (QECCs); sparse-graph quantum codes

I. INTRODUCTION

Quantum information processing (QIP) is an active research area with numerous applications, including quantum teleportation and quantum computing [1]. In order to perform an arbitrary quantum computation operation a minimum number of gates, known as universal quantum gates [1],[2]-[4], is needed. In this paper, we show that arbitrary single-qubit gate can be implemented based on single optical hybrid (OH)/Mach-Zehnder interferometer (MZI)/directional coupler (DC). We also show how to implement the deterministic CNOT gate based on OH/MZI/DC and highly nonlinear optical fiber (HNLF), which completes the implementation of arbitrary set of universal quantum gates in all-fiber technology. Two basic quantum circuits needed in quantum teleportation are Bell states preparation circuit and quantum relay. We also show how to implement them in integrated optics. The QIP, unfortunately, relies on delicate superposition states, which are sensitive to interactions with environment, resulting in decoherence. Moreover, the quantum gates are imperfect and the use of quantum error correction coding (QECC) is essential to enable the fault-tolerant computing and to deal with quantum errors [1]. In our recent paper [5] we proposed several structured quantum LDPC codes based on the balanced incomplete block designs (BIBDs), which offer a number of advantages thanks to the sparness of their quantum check-matrix. We show that encoder/decoder for arbitrary quantum LDPC code can be implemented in integrated optics as well.

II. INTEGRATED OPTICS IMPLEMENTATION OF UNIVERSAL QUANTUM GATES AND PAULI GATES

In what follows, the logical "0" is represented by a horizontal (H) photon $|H\rangle=[1\ \ 0]^T$ and the logical "1" is represented by a vertical (V) photon $|V\rangle=[0\ \ 1]^T$. An arbitrary single-qubit gate can be implemented in integrated optics based on OH, MZI or DC as shown in Figs. 1 (a)-(c), respectively. By expressing the power splitting ratio of OH as $k=\cos^2(\gamma/2)$ where angle γ is used to parameterize the power splitting ratio, the output qubit $[\psi_{H,o},\ \psi_{V,o}]^T$ is related to the input qubit $[\psi_H,\ \psi_V]^T$ by

$$\begin{bmatrix}\psi_{H,o}\\ \psi_{V,o}\end{bmatrix}=U\begin{bmatrix}\psi_H\\ \psi_V\end{bmatrix},\ U=\begin{bmatrix}\cos\left(\frac{\gamma}{2}\right)e^{j(\alpha-\beta/2-\delta/2)} & -\sin\left(\frac{\gamma}{2}\right)e^{j(\alpha-\beta/2+\delta/2)}\\ \sin\left(\frac{\gamma}{2}\right)e^{j(\alpha+\beta/2-\delta/2)} & \cos\left(\frac{\gamma}{2}\right)e^{j(\alpha+\beta/2+\delta/2)}\end{bmatrix}. \quad (1)$$

The matrix U in eq. (1) represents the matrix representation of an arbitrary single-qubit quantum gate according to the Z-Y decomposition theorem (see equation (4.12) in [1]). The same equation holds for MZI-based and DC-based single-qubit quantum gate (see Figs. 1(b,c)). By setting $\gamma=\delta=0$ rad, $\alpha=\pi/4$ and $\beta=\pi/2$ rad U-gate described by (1) operates as the phase gate; by setting $\gamma=\delta=0$ rad, $\alpha=\pi/8$ and $\beta=\pi/4$ rad the U-gate operates as $\pi/8$ gate; while by setting $\gamma=\pi/2$, $\alpha=\pi/2$, $\beta=0$ rad and $\delta=\pi$, the U-gate operates as Hadamard gate.

To complete the implementation of the following set of universal quantum gates {Hadamard, phase, $\pi/8$, CNOT}, the implementation of CNOT-gate is needed. The authors in [2] proposed the use of directional couplers to implement the CNOT-gate. However, in that proposal, the control output qubit is affected by input target qubit, which violates the definition of CNOT-gate operation (control qubit must be unaffected by target qubit) [1]. The CNOT gate from [2] operates correctly only with probability of 1/9, and is essentially a probabilistic gate. In Fig. 2 we show the deterministic implementation of CNOT-gate based on optical hybrid shown in Fig. 1 and HNLF. By using the quantum-mechanical description provided in [1], it can be shown that output control $|C_o\rangle=[c_{H,o}\ c_{V,o}]^T$ and target qubits $|T_o\rangle=[t_{H,o}\ t_{V,o}]^T$ are related to corresponding input qubits by:

$$\begin{bmatrix}c_{H,o}\\ c_{V,o}\\ t_{H,o}\\ t_{V,o}\end{bmatrix}=\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{bmatrix}}_{I\otimes H}\underbrace{\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}}_{K}\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{bmatrix}}_{I\otimes H}\begin{bmatrix}c_H\\ c_V\\ t_H\\ t_V\end{bmatrix} \quad (2)$$

$$=U_{CNOT}\begin{bmatrix}c_H\\ c_V\\ t_H\\ t_V\end{bmatrix},\ U_{CNOT}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

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The Kerr nonlinearity device in Fig. 2 performs the controlled-Z operation. In the absence of control c_V-photon the target qubit is unaffected because $H^2=I$ (identity operator). In the presence of control c_V-photon, thanks to the cross-phase modulation in HNLF, the target vertical photon experience the phase shift χL, where χ is the third order nonlinearity susceptibility coefficient and L is the HNLF length. By selecting appropriately the fiber length we obtain $\chi L=\pi$ and the overall action on target qubit is $HZH=X$, which corresponds to the CNOT gate action.

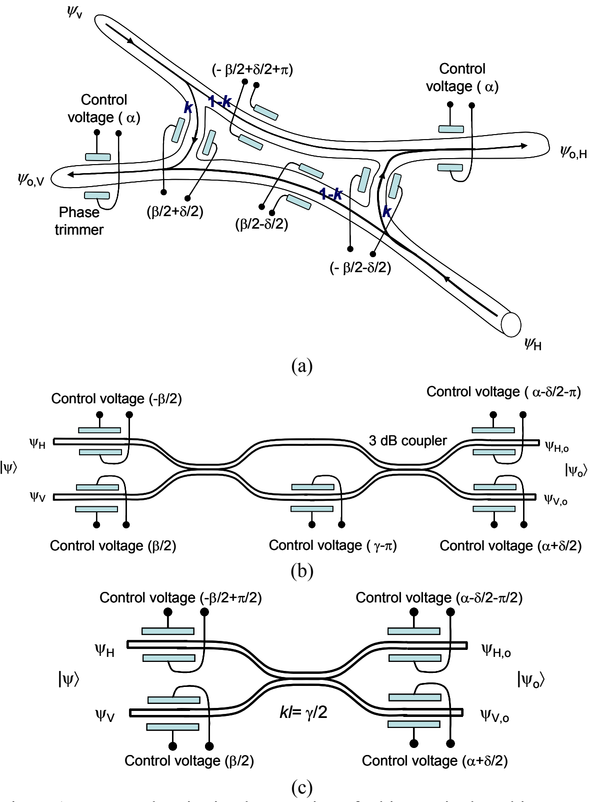

Figure 1. Integrated optics implementation of arbitrary single-qubit quantum gate based on single: (a) optical hybrid, (b) Mach-Zehnder interferometer and (c) directional coupler (k-the coupling coefficient, l-the coupling region length).

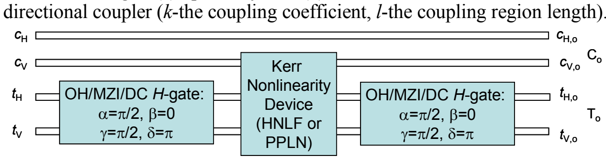

Figure 2. Implementation of CNOT gate based on optical hybrid and HNLF. OH/MZI/DC H-gate: OH/MZI/DC based Hadamard gate, PPLN: periodically poled LiNbO3.

With small modifications in (1) and by using similar approach to that described in Fig. 2, we can easily obtain the Barenco gate; while Deutsch gate can be obtained by employing three control qubits, instead of one used in Fig. 2.

By using the U-gate shown in Fig. 1 and by appropriately setting the phase shifts α, β, γ, and δ we can obtain the corresponding Pauli gates. The Y-gate is obtained by setting $\gamma=\pi$, $\beta=\delta=0$ rad and $\alpha=\pi/2$; the Z-gate is obtained by setting $\gamma=\delta=0$ rad, $\alpha=\pi/2$ and $\beta=\pi$; while the X-gate is obtained by setting $\gamma=\pi$, $\delta=0$ rad, $\alpha=\pi/2$ and $\beta=-\pi$.

III. Integrated Optics Implementation of Bell States Preparation Circuit and Quantum Relay

We further describe the implementation of Bell states preparation circuit based on OH/MZI/DC, which is shown in Fig. 3(a). The upper OH/MZI/DC circuit operates as Hadamard gate, while the rest of the circuit operates as CNOT gate. In Fig. 3(b) we describe how to implement the quantum relay based on Bell states preparation circuit (shown in Fig. 3(a)), Hadamard, controlled-X and controlled-Z gates described above. We employ the principle of differed measurement and perform corresponding measurements only in last intermediate node, which is a key difference with various quantum relay architectures described in [6]. The measurements circuits in Fig. 3(b) represent avalanche photodetectotors (APDs), which are used to detect the presence of c_V-photons in corresponding control qubits. The detection of c_V-photons triggers the application of required control voltages on phase trimmers to perform controlled-X and controlled-Z operation.

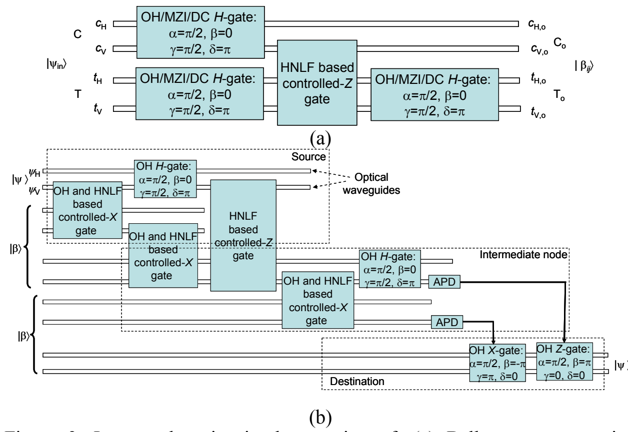

Figure 3. Integrated optics implementation of: (a) Bell states preparation circuit, and (b) quantum relay.

IV. Integrated Optics Implementation of Quantum LDPC Encoders/Decoders

The QECC encoders/decoders are essentially based on Pauli gates, whose implementation in integrated optics is already described in previous section. We have recently shown [5] that encoders and decoders for quantum LDPC codes can be implemented based on Hadamard and CNOT gates only. Therefore, arbitrary quantum LDPC encoder and decoder can be implemented in integrated optics based on OH/MZI/DC.

References

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