Transfer Matrix Analysis of Spectral Response of Even-Row Microring Resonator Arrays

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Abstract: The spectral response of even-row microring resonator arrays is analysed using a transfer matrix model. A simple 2×2 microring resonator array is firstly studied analytically, and the results are found to agree well with the FDTD simulation. Finally, we provide detailed results of complex spectral responses for larger array sizes.

Keywords: Integrated optics, microring resonator, transfer matrix model, finite-difference time-domain method

Introduction

Microring resonators are promising building blocks for future high density optical circuits [1]. To date, many studies have been performed based on multiple microring resonator systems in both series and parallel forms. Recently, microring resonator arrays (MRAs) have been proposed and studied, but these have mainly focused on the odd-row MRAs for filtering [2], pulse repetition rate multiplication and shaping applications [3]. A 4×*N* MRA has been demonstrated for programmable spectral-phase encoding (SPE) and decoding for wavelength-division-multiplexing (WDM)-compatible optical code-division multiple access (OCDMA) systems [4, 5]. In *M* series coupled microring resonators (i.e. an MRA with just one column), the morphology-dependent spectral resonances split into *M* higher-*Q* modes, which are defined as mode splitting [6]. However, there has been less attention on the theoretical study of the even-row MRAs. Hence, in this paper we study even-row MRAs, and show that the spectra are modulated by the presence of the coupled resonators and feedforward waveguides, rather than exhibiting mode-splitting.

Transfer Matrix Model

Figure 1(a) shows an MRA with *M* rows and *N* columns. Each column contains *M* coupled resonators and is connected to each of the other columns in the array via waveguides placed at the top and bottom of the array. Figure 1(b) gives the detailed representation of each column in the MRA. The array is also denoted as MRA(*M*,*N*) or an *M*×*N* MRA.

The transmission spectrum of an even-row MRA is calculated using conventional transfer matrix formalisms [2, 3]. The transverse transfer matrix of the nth column is denoted as U_n, and the transfer matrix of straight waveguides between the nth and (n-1)th columns is V_{n-1} with V_0=(1,0;0,1) assumed. After decompositions and iterations, normalized complex transfer functions at the drop and through ports can be written as (ξ_T; ξ_D) =(P_{22}; P_{12}), where $P=\prod_{n=1}^{N} V_{n-1}U_n$ is the total transfer matrix. $D=|\xi_D|^2$ is the normalized transmission spectrum at the drop port. Significant differences between the even-row and odd-row MRAs can thus be deduced. For example, every column of straight waveguides provides feedback paths for an odd-row MRA but feedforward paths for an even-row MRA. Mathematically, the complex optical propagation coefficient for the straight waveguide l_{n-1} cannot be distilled from V_{n-1} for the odd-row MRA. However, $P=(\prod_{n=1}^{N} l_{n-1})(\prod_{n=1}^{N} U_n)$ for an even-row MRA, where the product of every column of ring resonators is $\prod_{n=1}^{N} U_n$ and the product of every column of straight waveguides is $\prod_{n=1}^{N} l_{n-1}$. Finally, due to the reasons above, the normalized transmission spectrum at the drop port is not affected by l_{n-1} for an even-row MRA, while it is significant for an odd-row MRA.

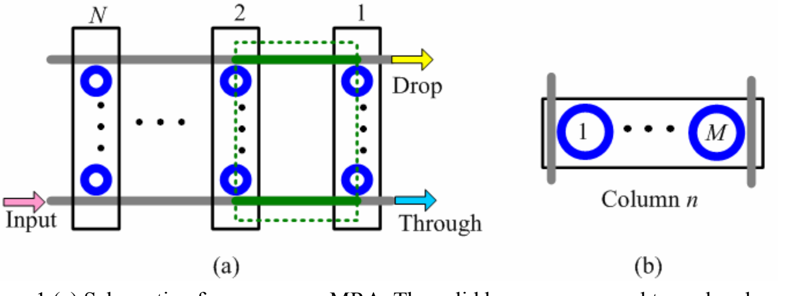

Figure.1 (a) Schematic of an even-row MRA. The solid boxes correspond to each column of M cascaded microring resonators and the dash box corresponds to straight waveguides between two adjacent columns. (b) Each column of M cascaded microring resonators.

Using the transfer matrix model above, transmission characteristics can be calculated for MRAs with arbitrary ring sizes and coupling coefficients. The values of the coupling coefficients are critical for the performance of microring resonators, strongly affecting the spectral shape and the transmission full width at half maximum. In the following investigations, ring sizes are assumed identical and the resonators are lossless. For simplicity, we denote θ and $\Delta\theta$ as the total and normalized round-trip phases in the resonator, respectively, with $\theta = \Delta\theta + 2m\pi$, where m is the resonance order. At resonance, it is assumed that $\Delta\theta = 0$. The relation between θ and radius R is given by $\theta = 4\pi^2 n_{eff} R / \lambda$, where n_{eff} is the effective reflective index of the waveguide and λ is the optical wavelength.

Analytical and Numerical Results

The simplest even-row MRA, i.e., the MRA(2,2), is firstly investigated using both the analytical transfer matrix method and a rigorous finite-difference time-domain (FDTD) simulation. The waveguide-resonator and inter-resonator coupling spectrum at the drop port is found as $D = 4\rho^2\sigma^2/\left(\rho^2+\sigma^2\right)^2$, where $\rho = \cos\theta - (2-k^2)/2$ and $\sigma = k^3/(2\sqrt{1-k^2})$. The solid line in Fig.2(a) is the transmission spectrum of MRA(2,2) with $k = 0.5$. The transmission spectrum of MRA(2,1) is also calculated for comparison. When the two resonators are series coupled as in MRA(2,1), mode splitting is clearly present. However, more complex modulated spectral features appear in MRA(2,2). The resulting zero transmission occurs in MRA(2, 2), where unity transmission occurs for MRA(2,1). 2D FDTD simulation [7] is adopted to validate the accuracy of the analytical model based on the transfer matrix method. A Gaussian pulse at a centre wavelength of 1.5μm is injected into the resonator system. The waveguide is chosen to be high index semiconductor material with air as the cladding. Thus, the refractive index difference is as high as 2.4 to provide a small radius of microring resonators. The radius of microring resonators is chosen to be 2μm with a waveguide width of 0.2μm. The widths of inter-resonator and waveguide-resonator gaps are chosen as 0.05 and 0.10μm, respectively. Fig. 2(b) gives numerical results of the spectral response of the MRA(2,2) structure for wavelengths between 1.4 and 1.6 μm. This response shows good agreement with the results obtained using the analytical model.

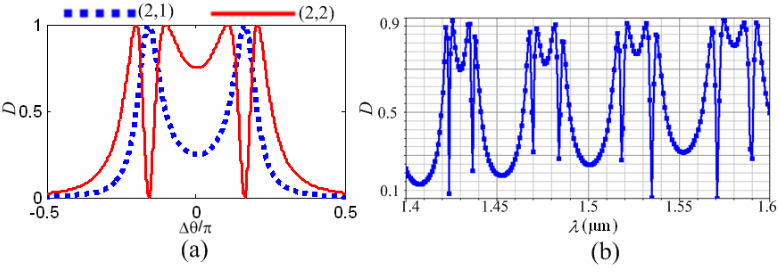

Figure. 2 (a) Transmission spectral of MRAs (2,1) and (2,2), respectively. (b) Transmission spectrum of MRA(2,2) using FDTD simulations.

We further study the spectral performance of even-row MRAs for larger array sizes. The coupling coefficient k is firstly assumed to be 0.5 for each coupler. Fig. 3(a)-(e) shows the transmission spectra for MRAs (2,3), (2,5), (2,6), (2,7) and (2,9), respectively. Fig. 3(f) shows the transmission spectral plotted in contour form, where zero transmissions are black. It can be seen that strong modulation of the spectra occurs in the spectral regions where originally mode splitting has occurred, and that the spectral modulation becomes very strong. To characterise this spectral responses, the number of zeros is denoted as M_T. Fig. 3(g) gives the spectral response when M is increased to 4, and shows that more zeros appear. Generally, M_T is observed to equal $M(N$-1), with N-1 zeros near each splitting mode of MRA (M,1), as shown in Fig.3(f) when N varies from 1 to 5. However, two or more zeros merge when the coupling coefficient is larger than some specific values. The transmission spectra are calculated as a function of the coupling coefficient k as shown in Fig. 4. For the case of MRA(2,6), there is just one region at $\Delta\theta = 0$ where zeros merge as shown in Fig.4(a). As k increases, the most inner zeros merge when k=0.5 and the following inner zeros merge when k=0.87. However, for the case of MRA(4,6), there are three regions where zeros merge as shown in Fig.4(b). Zeros at $\Delta\theta = 0$ merge at the same k as that of the MRA(2,6). Zeros at two outer symmetrical regions merge at k=0.58 and 0.91 sequentially.

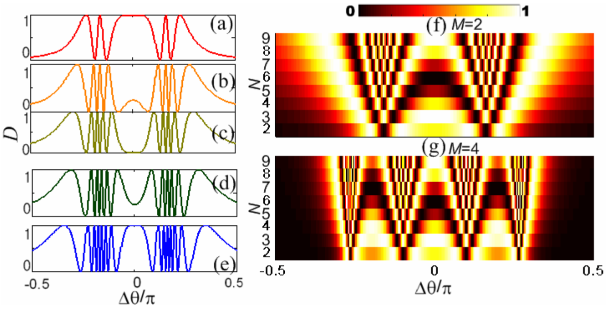

Figure 3. (a)-(e) Transmission spectra of MRAs (2,3), (2,5), (2,6), (2,7) and (2, 9), respectively.

(f) Contour plot of transmission spectra corresponding to MRAs(2,2:9).

(g) Contour plot of transmission spectra corresponding to MRAs(4,2:9).

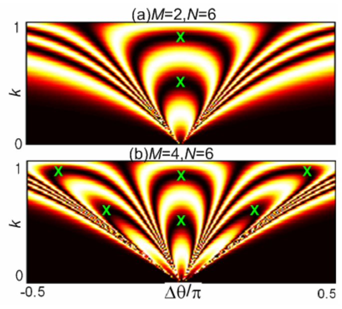

Figure 4. (a), (b)Contour plots of transmission spectra of MRAs (2,6) and (4,6) as a function of the coupling coefficient, respectively. 'x' points correspond to values of k where zeros merge.

Conclusion

To conclude, we have analysed the spectral response of even-row MRAs, and a large number of zeros is found to appear in the spectra. Thus the spectra are modulated, which is caused by combination effects of resonator interactions and waveguide feedforwards, compared to mode-splitting in series coupled microring resonators. This kind of modulated spectra may have potentials in spectrally dependent sub-system applications. By tuning each column's effective index and coupling coefficients, it would be possible to control the response of even-row MRAs, which allow very large numbers of spectral amplitude codes to be generated for OCDMA systems.

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