Phase velocity estimation of a modulation wave in a quasi-velocity-matched electro-optic phase modulator using electro-optic sampling system

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Abstract - *Direct observation of an electric field distribution on a resonant microstrip line (width:0.5 mm, height:0.5 mm, substrate: stoichiometric LiTaO$_3$) has been carried out using electro-optic sampling system. The phase velocity of* $5.4 \pm 0.25 \times 10^7$ *m/s has been estimated from a resonance frequency and an observed wavelength of a standing wave.*

Introduction

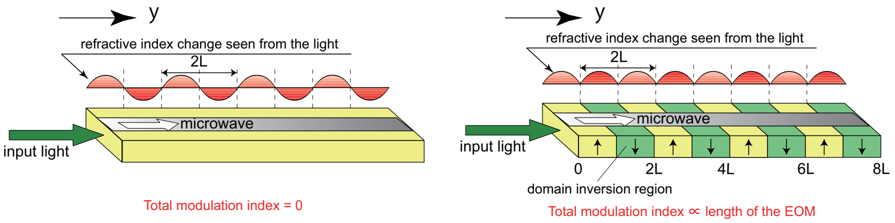
Wide expansion of an optical spectrum by deep phase modulation at a high modulation frequency is essential technique for many electro-optic (EO) light controlling fields, such as ultrashort pule generation[1], comb generation [2], time-to-space mapping of an optical signal[3] and so on. At a high modulation frequency region, traveling-wave EO modulator (EOM) has been widely used because of a long interaction length between an optical wave and a modulation wave. However, since there is so-called velocity mismatching between the optical wave and the modulation wave in the EO crystal, when the modulation frequency is as high as over ten gigahertz, the modulation index varies periodically as the interaction length increases.

To achieve deep modulation at high frequency, quasi-velocity-matching (QVM)[4] with periodic domain inversion of a ferroelectric crystal is effective technique. In the QVM technique, phase velocity of the modulation wave should be known so that the period of the periodic domain inversion can be determined. We calculated the modulation phase velocity through the empirical formula using relative permittivity [5, 6] which is measured for other LiTaO$_3$ samples at other modulation frequency so far. The modulation phase velocity can also be estimated from the $|S_{11}|$ spectrum of the resonant microstrip line, however the estimation is largely influenced by the electrode implementation (for example, the electrical contact between the microstrip line and the connector) and is inaccurate.

In this paper, we estimate the phase velocity of the modulation microwave of 16 GHz on the basis of the direct observation of the electric field distribution in the resonant microstrip line using EO sampling system[7]. Based on the EO sampling system, we can measure the wavelength of the standing wave precisely. The phase velocity v_m is calculated from measured wavelength of the standing wave λ_m and resonance frequency f_m as $v_m = \lambda_m f_m$.

Quasi-velocity-matching

Figure 2 (a) and (b) shows the schematic of the traveling-wave EOM and QVM-EOM, respectively. To avoid unnecessary complexities, we employ a one-dimensional analysis



(a) Traveling-wave EOM. (b) QVM-EOM.

Figure 1: (a) Traveling-wave EOM, (b) QVM-EOM with periodic domain inversion.

for our devices. For a traveling-wave EOM, if the y axis is the direction in which the optical and modulation wave propagate in a virgin (single domain) EO crystal, the variation of the refractive index induced by an electric field $E_m \cos(\omega_m t)$ is obtained at y as

$$\Delta n(y,t_0) = n_m \cos(\omega_m t_0 - \frac{\pi}{L} y), \tag{1}$$

where $n_m = \frac{1}{2} n_e^3 \gamma_{33} E_m$ is the amplitude of index changes, n_e is the extraordinary refractive index of the crystal, γ_{33} is the EO coefficient of the crystal, and $\omega_m = 2\pi f_m$. L is a half-period of the domain inversion for QVM given by $L = [2 f_m (v_m^{-1} - v_o^{-1})]^{-1}$, where v_m is the phase velocity of the modulation wave, and v_o is the group velocity of the optical wave. Here, we assume that the optical wave arrives at point $y = 0$ at time $t = t_0$.
For $v_o > v_m$, the phase shift of light at the position y is expressed as

$$\theta = \frac{2\pi}{\lambda} \int_0^y \Delta n(y,t_0) dy = \Delta\theta \cos(\omega_m t_o - \frac{\pi}{2L} y), \tag{2}$$

where,

$$\Delta\theta = \frac{4L}{\lambda} n_m \sin(\frac{\pi}{2L} y). \tag{3}$$

Since the traveling-wave EOM has so-called velocity mismatching between modulation wave and optical wave, the modulation index $|\Delta\theta|$ becomes a periodical function of y with period $2L$.
When a traveling-wave EOM has a suitable domain-inverted half-period of L, QVM occurs and accordingly a large modulation index is achieved. In such a situation, phase retardation $\Delta\phi(x,t_0)_M$ given to the light passing through the length of $2ML$ in a periodically domain-inverted crystal as shown in Fig. 2(b) is expressed as

$$\Delta\phi(x,t_0)_M = \frac{2M\pi}{\lambda} \int_0^{2L} g(x,y) \Delta n(y,t_0) dy, \tag{4}$$

where λ is the vacuum wavelength of the light, $g(x,y)$ is -1 and 1 for domain-inverted regions and non-domain-inverted regions, respectively.
Figure 2 shows modulation indices of EOM with (a) QVM-EOM, and (b) traveling-wave EOM (velocity mismatching). The QVM modulation index is almost proportional to the interaction length though it is lower than perfect velocity-matched condition by a factor of $2/\pi$.

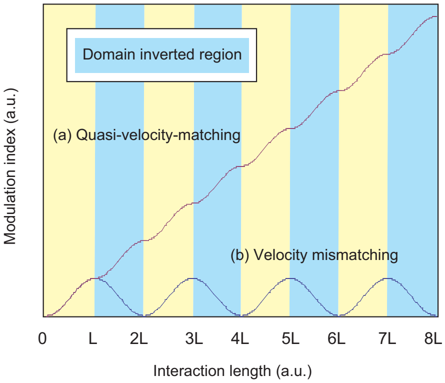


Figure 2: Modulation indices of EOM. (a) QVM-EOM with periodic domain inversion, and (b) traveling-wave EOM (velocity mismatching).

Estimation of the phase velocity of the modulation wave

The half-period of the domain inversion for the QVM is calculated as

$$L = \frac{1}{2f_m(\frac{1}{v_m} - \frac{1}{v_o})}, \tag{5}$$

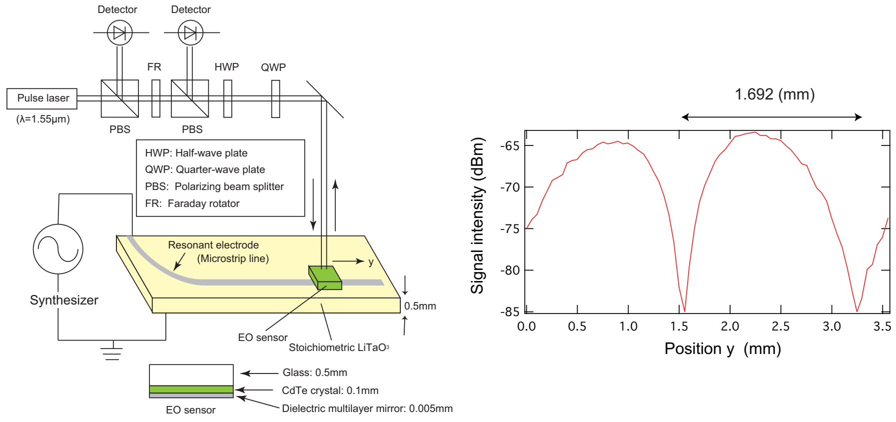
and it is obvious that the modulation efficiency is directly influenced by the estimation accuracy of the phase velocity of the modulation wave, v_m, and the group velocity of the optical wave, v_o. Optical group velocity can be measured in relatively high accuracy. It is important to estimate the phase velocity of the modulation wave precisely to optimize the operation of the QVM-EOM.

Figure 3(a) shows experimental setup. We fabricated the resonant EOM with stoichiometric $LiTaO_3$ substrate. The height of the $LiTaO_3$ crystal was 0.5 mm. A silver microstrip line of 0.5 mm width was evaporated on the crystal. The resonant frequency was 15.91 GHz. The pulsed fiber laser (repetition rate: 40 MHz) was used for probe pulses. The beam spot size was about 80 μm. The CdTe crystal was used for the EO sensor. We scanned the probe beam in the y direction and measured electric field profile of the resonant standing wave.

Figure 3(b) shows a typical experimental result. From the amplitude distribution of the standing wave, the resonant half- wavelength was determined as $\lambda_m/2 = 1.7$ mm. Consequently, the phase velocity can be calculated as $v_m = 5.4 \times 10^7$ m/s. By measuring λ_m for four times at different position of the microstrip line, we estimated the phase velocity as $v_m = 5.4 \pm 0.25 \times 10^7$ m/s. The half-period of domain inversion of $L = 2.95$ mm can be derived from the phase velocity of $v_m = 5.4 \times 10^7$ m/s. If the $v_m = 5.4 \times 10^7$ m/s is correct value rather than the $v_m = 5.3 \times 10^7$ m/s, which is used in our former experiment, improvement of about 10% of the modulation index can be expected.

Conclusion

We have estimated the modulation phase velocity in the QVM-EOM by direct observation of the electric field of the resonant standing wave using EO sampling system. The phase velocity of $v_m = 5.4 \pm 0.25 \times 10^7$ m/s has been estimated.



(a) (b)

Figure 3: (a)Experimental setup. (b) Measured electric field for modulation frequency of 15.91 GHz.

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