Noise and resolution in IO interferometric sensing

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Abstract: *The paper presents a general theory for sensing devices, relating noise and device parameters to resolution of modal index changes. The theory is applied to optimise the length of a few integrated optics sensing devices, being a Mach-Zehnder interferometers and two Fabry-Perot implementations. The results enable the determination of the maximum attainable resolution, and show the crucial importance of loss.*

Introduction

Integrated optics (IO) refractometric sensors are suitable candidates for the accurate detection of parameters like index changes, layer thickness changes and displacements. The advantages of such sensors are the following:

- Diffractionless propagation,
- The possibility for in-situ detection,
- The possibility for compact integration of all kind of functions, and also of
- Multi-sensing arrays for multi-parameter detection,
- The possibility for sensor set-ups that can not or hardly be realized with classical optics components.

Related to the latter item we note that indeed a large number of different typically IO sensing devices have been proposed and realized, based on for example mode coupling [1], modal field changes [2], grated waveguides (WGs) [3], photonic crystals [4] or surface plasmon resonance (SPR) [5,6]. In addition quite a number of IO implementations of classical optics instruments have been studied in detail, like Mach-Zehnder [7], Young [8,9] and Fabry-Perot interferometers [10,11], abbreviated by MZIs, YIs and FPIs, respectively. An overview can be found in [12].

It is the aim of this paper to present a theoretical frame work describing the relation between device parameters, including propagation loss, and modal index resolution, in the presence of detection noise. For parameter reduction it appears to be convenient to introduce the so-called normalized specific resolution (NSR). The presented theory is used for parameter optimization of a number of IO implementations of classical optics devices, being a balanced MZI, a symmetric and an asymmetric FPI. The crucial role of modal propagation loss for the attainable resolution will be shown. The paper ends with conclusions.

Effect of noise on the effective index resolution

We consider sensing set-ups in which the change of the modal index *N*, which results from an index change in the sensing section is derived from the measured transmittance, *T*. *T* is measured as a function of some scanning variable *s*, which varies in time with a modulation frequency. Such a variable could be the angle of incidence in SPR, the modulation voltage applied in both branches of an MZI [7], a spatial coordinate along which the power distribution is monitored, in a YI [8,9], or the wavelength in an FPI. The advantages of utilizing such a scanning variable is that large amount of data points at one scan enable to correct for drift, being defined as the effect of disturbing factors with frequencies appreciably lower than the modulation frequency and reduces the effects of noise (frequencies appreciably higher than the modulation frequency) in the output power by fitting the response curve. On fitting the analytical expression of the response curve (in terms of *s* and the modal index *N*) should be known, by theoretical or experimental means, except possibly for a few adjustable parameters (next to *N*).

In the paper we will focus on two types of noise:

1. noise which is proportional to the output power , such as noise resulting from fluctuations in source power, noise introduced by mechanically unstable parts of the device like fiber to chip couplers or noise corresponding to scattered light in the output signal.
2. noise of which the value is independent of the output power such as detector dark current noise.

Hence, the assumed noise in the output power, P_{out}, is given by:

$$\delta P_{out}^{total} = \delta P_{out} + \delta P_{dark} . \qquad (1)$$

The first term at the right hand side of equation 1 is proportional to P_{out}, which may consist of contributions from both functional output and re-captured stray light. The second term at the right hand side of equation 1 denotes noise related to the dark current, which is assumed to be independent of the output power. The first term in the above is usually the largest unless the output power is relatively low, in for example devices with large losses.

Rewriting equation 1 in terms of the dimensionless quantity transmittance defined by

$$T \equiv P_{out} / \bar{P}_{in} ,$$

with $\bar{P}_{in}$ the time-averaged input power, it follows:

$$\delta T^{total} = \delta T + \delta\tilde{T} \equiv (\delta P_{out} + \delta P_{dark}) / \bar{P}_{in} . \quad (2)$$

In the below normal (Gaussian) distributions of the noise around zero is assumed, with standard deviations of $T\sigma_T$ and $\tilde{\sigma}_T$, corresponding to the terms δT and $\delta\tilde{T}$, respectively, in equation 2. Note that

$\tilde{\sigma}_T$ decreases if P_{in} increases. In most practical situations things can be arranged such that $\tilde{\sigma}_T \ll \sigma_T$.

In case that inaccuracy in the scanning parameter plays a relevant role (which should of course possibly be avoided) a term $\delta T^s = (\partial T / \partial s)\delta s$ has to be added to equation 2. In the below it is assumed that the accuracy of the set-up is such that its effect on the modal index resolution can be neglected, as well as effects of the drift.

The measured transmittance on varying *s* is given by:

$$T_{m,l} = T_{c,l} + \delta T_l \, , \qquad (3)$$

where T_c indicates the correct curve, corresponding to a measurement with zero noise, δT denotes the noise and the subscripts *l* indicate the sampling points corresponding to *s*. The total number of data points is *Q*. In the below it is assumed that, from the measurements, only the modal index, *N*, has to be obtained. In practical situations generally more parameters have to be determined, in order to correct for drift, in for example laser power. It can be made plausible that theses additional fitting parameters hardly affect the error in the fitting of *N*.

For the obtained fitted response curve, T_f, the following holds:

$$T_{f,l} = T_{c,l} + \left(\partial T_{c,l} \big/ \partial N_f\right)\Delta N_f \qquad (4)$$

where ΔN_f is the error as a result of noise. The fitting procedure is assumed to consist of minimizing the following function of N_f:

$$F(N_f) = \sum\nolimits_{l=1}^{Q} (T_m - T_f)_l^2 \, . \qquad (5)$$

Equation 5 can be re-written by substitution of equations 3 and 4, leading to:

$$F = \sum\nolimits_{l=1}^{Q} \left[\delta T_l - \left(\partial T_{c,l} \big/ \partial N_f\right)\Delta N_f \right]^2 . \qquad (6)$$

The minimum of *F* as a function of N_f follows from:

$$F'(N_f) = \sum_{l=1}^{Q} 2\left(\frac{\partial T_{c,l}}{\partial N_f}\right)\left(\left(\frac{\partial T_{c,l}}{\partial N_f}\right)\Delta N_f - \delta T_l\right) = 0 \, . \qquad (7)$$

From equation (7) it follows (omitting the subscripts *c* and *f* in the derivatives, for simplicity):

$$\Delta N_f = \sum_{l=1}^{Q} (\partial T_l / \partial N)\delta T_l \Bigg/ \sum_{l=1}^{Q} (\partial T_l / \partial N)^2 \, . \qquad (8)$$

For an ensemble of such N_f determinations the standard deviation is given by:

$$\sigma_N = \frac{\sqrt{\sum\nolimits_{l=1}^{Q} (T_l^2 \sigma_T^2 + \tilde{\sigma}_T^2)(\partial T_l / \partial N)^2}}{\sum\nolimits_{l=1}^{Q} (\partial T_l / \partial N)^2} \, . \qquad (9)$$

Here we have used equation 2 and repeatedly the fact that the resulting standard deviation on adding two stochastic quantities is given by the square root of the sum of the two squared standard deviations involved. Neglecting the effect of variations in the terms under the summations in equation 9 one can state roughly that the resulting standard deviation is proportional to $1/\sqrt{Q}$, and that on adding data points the value of σ_N is always reduced, even if $|\partial T / \partial N|$ is small. It is also clear from equation 9 that in general the inclusion of data corresponding to large values of $|\partial T / \partial N|$ will lead to a strong decrease in the standard deviation. Note that the standard deviation σ_N does not depend implicitly on the scanning variable *s*. The advantage of using a scanning variable lies mainly in the fact that data points with large $|\partial T / \partial N|$ values are part of the response curve.

Using equation 9 we can now define the modal index resolution quantitatively by $\rho \equiv 1/\sigma_N$.

Device optimization for a number of integrated optics sensing devices

In this section the optimization of a few IO sensing devices will be discussed. The considered devices are a balanced, modulated MZI, a symmetric and a-symmetric FP-cavity. A general approach to optimize the resolution of these devices is presented. It is assumed that only functional device output is captured by the detector (i.e., no stray light), propagation loss is taken into account and, for simplicity, only a single data point, corresponding to the maximum value of $|\partial T / \partial N|$, is assumed in the expressions for the modal index resolution. The true resolution can be determined roughly from it by multiplication with $\sqrt{Q}$, with *Q* the number of sampling points.

In the *balanced MZI* it is assumed that the loss in both branches is equal. The transmittance of the device is given by:

$$T_{bMZI} = 0.5[1 + \cos(\varphi_N + \varphi_s)]e^{-\alpha L} , \qquad (10)$$

with $\alpha \equiv 2k_0 N_{im}$, N_{im} being the imaginary part of the modal index, $\varphi_N \equiv k_0(N - N_r)L$, with k_0 the wavenumber, N_r the modal index of the reference branch, and *L* is the length of the sensing section. The phase shift φ_s is due to electro-optic modulation (see inset of figure 1), and can for example be varied between 0 and 2π. In the above we neglected the assumed small losses in splitting and modulation sections. With equation 9, and using a single data point (at $\varphi_N + \varphi_s = \pi / 2$) it follows:

$$\sigma_N = \frac{1}{k_0 L}\sqrt{\sigma_T^2 + (2\tilde{\sigma}_T)^2 e^{2\alpha L}} \, . \qquad (11)$$

In the below we will introduce the so-called normalized specific resolution (NSR), *η*, defined via:

$$\rho (\equiv 1/\sigma_N) = \eta k_0 / (\alpha \sigma_T) \, . \qquad (12)$$

It can be shown that *η*, which is device specific, depends generally only on the normalized length, αL (rather than on both α and *L*), other device parameters (if any; like reflectance in a FP cavity), the ratio

$\sigma_T / \tilde{\sigma}_T$ and the used data points of the response curve.
From equation 11 it follows that minimum of σ_N, as a function of the device length, occurs at a normalized length αL_{opt} defined by:

$$(\sigma_T / \tilde{\sigma}_T)^2 = 4(\alpha L_{opt} - 1)e^{2\alpha L_{opt}}, \qquad (13)$$

leading, after substitution into equation 11, to

$$\sigma_N = \frac{\alpha \sigma_T}{k_0 \eta_{opt}}, \quad \eta_{opt} = \sqrt{\alpha L_{opt}(\alpha L_{opt} - 1)}. \qquad (14)$$

In figure 1 plots as a function of $\sigma_T / \tilde{\sigma}_T$ are given of αL_{opt}, obtained from equation 13 by taking the inverse function (Matlab function *lambertw*), the NSR, η_{opt}, according to equation 14, and the transmittance (for the considered single data point).

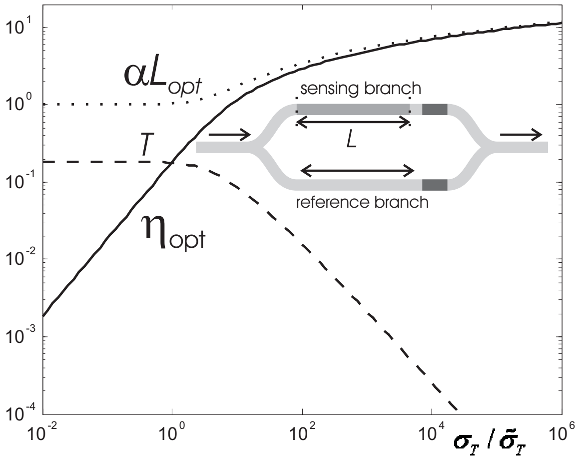

Fig. 1: Optimized NSR, transmittance and optimised nomalised length of a balanced MZI as s function of the ratio $\sigma_T / \tilde{\sigma}_T$ (see text). The inset shows a schematic lay out, including modulator sections [7].

For large $\sigma_T / \tilde{\sigma}_T$ values the exponent in equation 13 dominates, i.e., $\alpha L_{opt} >> 1$, and the optimum length can be approximated as follows:

$$\alpha L_{opt} = \ln\left[\frac{\sigma_T}{2\tilde{\sigma}_T\sqrt{\alpha L_{opt} - 1}}\right] \approx \ln\left(\frac{\sigma_T}{2\tilde{\sigma}_T}\right). \qquad (15)$$

From the above and equation 13 it follows:

$$\eta = \sqrt{\alpha L_{opt}(\alpha L_{opt} - 1)} \approx \alpha L_{opt} \approx \ln[\sigma_T / (2\tilde{\sigma}_T)], \; (16)$$

where use was made of the fact that $\alpha L_{opt} >> 1$.
The transmittance of a *symmetric FPI* is given by:

$$T_{sFP} = \frac{(1-R)^2 e^{-\alpha L}}{(1-R')^2(1 + F' \sin^2 \varphi_{N,\lambda})}, \qquad (17)$$

with L the cavity length, $\varphi_{N,\lambda} \equiv k_0 LN$, R the modal reflectance at the end-faces, $R' \equiv R \cdot \exp(-\alpha L)$ and $F' \equiv 4R'/(1-R')^2$. It is assumed in the above that there is no power loss at the end-faces, which are both assumed to be equal. Without losing generality, it is assumed that the wavelength is the scanning variable. Note that the above expression applies equally well to a ring resonator, with a circumference of 2L, coupled symmetrically to two WGs (see inset of figure 2).
Device optimization has been done numerically in the following way. It is assumed, for the moment, that noise due to the dark current can be neglected, i.e., the term with $\tilde{\sigma}_T$ in equation 9 is assumed to vanish. From the results presented below it appears that things can be arranged such that it plays only a minor role for the obtainable resolution in both symmetric and asymmetric FPIs. For a given reflectance, R, the value of $\varphi_{N,\lambda}$, for which $|\partial T / \partial N|$ has a maximum, is determined numerically using equation 17. Next, the normalized optimum device length, αL_{opt}, has been calculated numerically by maximizing the NSR, η_{opt}, with the Matlab *fminsearch*-function. The resulting values of αL_{opt}, η_{opt} and the transmittance, T, as functions of the reflectance are given in figure 2.

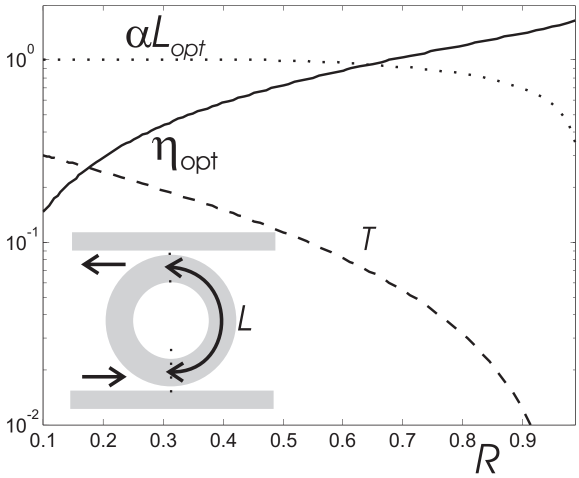

Fig. 2: η_{opt}, transmittance and optimised nomalised length of a symmetric FPI as s function of the reflectance. The inset shows a schematic lay out of the device.

The modal transmittance of the considered *asymmetric FP cavity*, with one fully reflecting end facet, is given by:

$$T_{aFP} = \frac{R - 2r\cos(2\varphi_{N,\lambda})e^{-\alpha L} + e^{-2\alpha L}}{1 - 2r\cos(2\varphi_{N,\lambda})e^{-\alpha L} + Re^{-2\alpha L}}, \; (18)$$

where L is the cavity length, $\varphi_{N,\lambda} \equiv k_0 LN$, $\alpha \equiv 2k_0 N_{im}$, $R(\equiv r^2)$ the modal reflectance at the input of the device and r is the corresponding reflection coefficient. Proceeding as above for the symmetric FP cavity, also assuming that there is no dark current noise and a single data point (corresponding to the maximum of $|\partial T / \partial N|$), the parameters αL_{opt}, η_{opt} and T have been determined numerically as a function of the reflectance. The result is given in figure 3.

Note that the considered structure is functionally

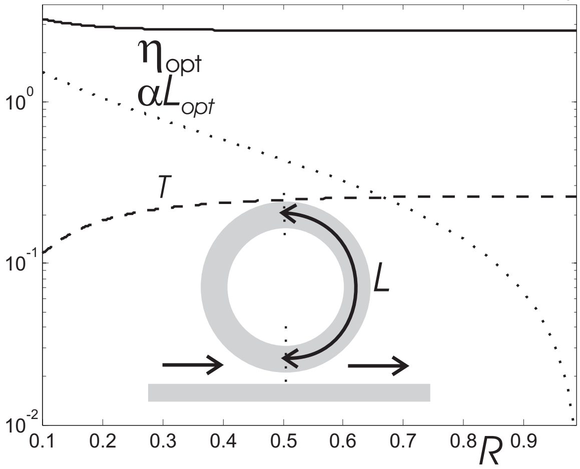

Fig. 3: η_{opt} , transmittance and optimised nomalised length of an asymmetric FPI as s function of the reflectance. The inset shows a schematic lay out of the device.

equivalent to a ring resonator, coupled to a single WG (see inset of figure 3).

Considering the graphs of the three considered devices the following main conclusions can be drawn:

- the length-optimized NSR, η_{opt} , for a MZI is determined by the ratio $\sigma_T / \tilde{\sigma}_T$, and can take on large values if this ratio is large, whereas
- its value is limited in both symmetric and asymmetric FPI to values of η_{opt} ~1 and 3, respectively.
- In the considered FPIs the effect of dark current noise is relatively small if $\sigma_T / \tilde{\sigma}_T \gtrsim 100$ and $T \gtrsim 0.01$ (for example), as can be seen with equation 9. The latter leads to the requirement $R \lesssim 0.9$ for the symmetric FPI (see figure 2).

The length-optimized resolution can be calculated from the presented η_{opt} curves with equation 12. Note, that, quite general, the absorption α plays a crucial role for this quantity

Concluding remarks

The paper presents theory of the relation between the resolution for modal index changes and noise and device parameters for a number of IO sensing devices, assuming a scanning variable, like the wavelength. The theory is applied on a balanced MZI, and a symmetric and asymmetric FPI, to calculate the length-optimised so-called normalised specific device resolution (NSR), η_{opt} , assuming that only a single data point of the response curve is used in the measurement. The NSR depends on device parameters, but not explicitly on α, the power loss coefficient, and k_0 , the wavenumber. From the NSR the resolution for modal index changes, ρ, can be calculated according to $\rho(\equiv 1/\sigma_N) = \eta k_0 /(\alpha \sigma_T)$, with σ_T the standard deviation in the transmittance and σ_N is the resulting standard deviation in the effective index.

From the results the following can be concluded:

- Loss is a crucial factor for the maximum attainable device resolution,
- The maximum attainable NSR of the balanced MZI can be large and limited by the presence of so-called dark current noise, whereas,
- Its value is always limited to η ~1-3 for considered FPIs.

As mentioned, for simplicity only a single data point of the device response curves was considered in the presented results. But, it is expected that the general features of the presented graphs, and also the main conclusions hold as well in case of considering a (much) larger number of data points (leading normally to higher resolution).

The presented results may be of help to select an IO sensing device for a certain application, although other properties may also be quite relevant, such as:

- Compactness,
- Sensitivity for temperature fluctuations,
- Robustness against technological errors, and
- Accuracy and range of the scanning variable.

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