

# Enhanced Transmission in Dynamically Tuned Optical Microresonator Possessing Highly Dispersive Media

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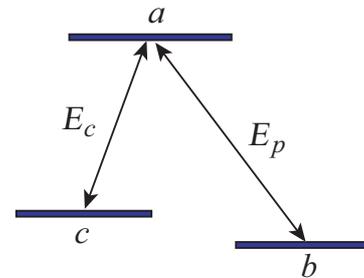
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**Abstract:** We present a theoretical study of an optical microresonator system which contains a electromagnetically induced transparency medium within the resonator. We find that a time-dependent tuning of the dispersive properties of the resonator medium results in an enhanced transmission spectrum.

## Introduction

Many interesting optical phenomena, such as electromagnetically induced transparency (EIT) and ultra-slow light propagation have been observed in highly dispersive media [1, 2]. Additionally, the introduction of a highly dispersive media into an optical cavity results in dramatic narrowing of spectral features as well as enhancing cavity lifetimes [3, 4, 5].

At the heart of EIT is a resonance condition established by a quantum interference effect known as coherent population trapping [6]. Figure 1 is a schematic of one such atomic arrangement known as the  $\Lambda$  scheme which exhibits EIT. The system consists of three atomic energy levels  $a$ ,  $b$  and  $c$ , where  $b$  and  $c$  are long-lived lower levels. EIT in this system is observed by applying two electromagnetic fields to the medium, a control field  $E_c$  and a probe field  $E_p$ . When the control field is tuned to the resonance frequency of the  $ac$ -transition, a transparency window opens for the probe field if the two-photon resonance condition is fulfilled. (Throughout this work we assume that the probe pulse is weak in comparison to the control field.) When two-photon resonance is satisfied, the single absorption peak in  $\text{Im}\{\varepsilon\}$  splits into two peaks, dashed and solid line respectively in Figure 2(b), that are separated symmetrically on the  $ab$ -transition resonance frequency,  $\omega_{ab}$ . The two peaks are also separated spectrally in a magnitude equal to the control field Rabi frequency,  $\Omega_c = p_{ac}E_c/\hbar$ , where  $p_{ac}$  is the dipole moment of the  $ac$ -transition. Since the absorption is near zero around  $\omega_{ab}$ , a probe field with that frequency is not absorbed as it propagates through the EIT medium. However, the frequency region around the transmission window is highly dispersive as shown by the solid line in Figure 2(a). It is this extreme dispersion which accounts for the line-width narrowing and enhanced lifetimes in a resonator cavity which contains this type of medium. In fact, as the dispersion increases, the line-widths increasingly narrow and the cavity lifetimes lengthen. However, simultaneously, the system becomes bandwidth limited as the absorption peaks move closer together. To circumvent this



**Fig. 1:** Schematic of the energy levels in a three level  $\Lambda$  system.

effect, one could imagine that a dynamic change in  $\Omega_c$  could allow an increased performance of the resonant cavity system.

## Calculation method

Recently, we described a finite-difference time-domain (FDTD) technique capable of modelling EIT [7] and incorporating a dynamic control field Rabi frequency. The algorithm utilized an implementation of the auxiliary differential equation (ADE) method [8] and a representation of the dielectric constant which is connected to experimental parameters, such as  $\Omega_c$ . Such an implementation of the FDTD method allows EIT media and arbitrary dielectric structures to occupy the same computational domain, a task not easily realizable in other calculations methods, such as the Maxwell-Bloch algorithm [9].

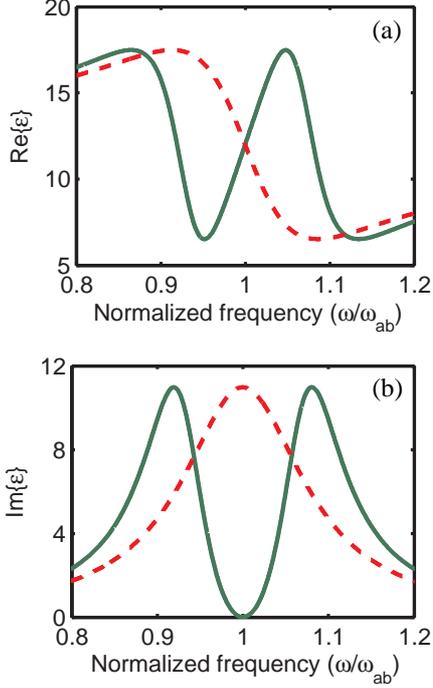
The susceptibility of a three-level EIT medium can be expressed as [10]

$$\chi(\Delta) = -\frac{N|p_{ab}|^2}{\varepsilon_0\hbar} \frac{\Delta + i\gamma_{bc}}{(\Delta + i\gamma_{bc})(\Delta + i\gamma_{ab}) - \Omega_c^2/4}, \quad (1)$$

where  $\Delta = \omega - \omega_{ab}$  is the probe beam detuning from the resonance frequency  $\omega_{ab}$  of the  $ab$ -transition,  $\gamma_{ab}$  is the loss due to the  $ab$ -transition,  $\Omega_c$  is the Rabi frequency of the control field,  $\gamma_{bc}$  is the decoherence rate,  $N$  is the density of excitation centers, and the  $p_{ab}$  is the dipole moment of the  $ab$ -transition. Assuming  $\chi \ll 1$ , we used a pole expansion to arrive at an exact, complex two-pole representation of the relative permittivity:

$$\varepsilon = \varepsilon_b + \sum_{l=1}^M \left( \frac{A_l}{B_l + i\omega} \right), \quad M = 2, \quad (2)$$

where  $\varepsilon_b$  is the background dielectric constant and  $A_l$



**Fig. 2:** (a) Real and (b) imaginary parts of the permittivity of an EIT medium when the control field is on (solid) or off (dashed).

and  $B_l$  are complex coefficients given by

$$A_l = \frac{iN|p_{ab}|^2}{2\varepsilon_0\hbar} \left\{ -1 + i^{(2l)} \frac{(\gamma_{bc} - \gamma_{ab})}{[(\gamma_{bc} - \gamma_{ab})^2 - \Omega_c^2]^{1/2}} \right\}, \quad (3)$$

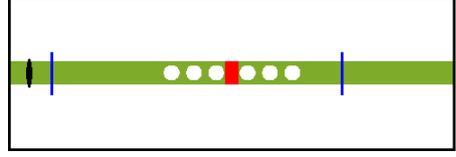
and

$$B_l = -i\omega_{ab} + \frac{1}{2} \left\{ \gamma_{bc} + \gamma_{ab} + (-1)^l [(\gamma_{bc} - \gamma_{ab})^2 - \Omega_c^2]^{1/2} \right\}. \quad (4)$$

The standard FDTD implementations are then performed in combination with the ADE method [8] to derive the field-update equations. In all, the update algorithm involves only one extra step since a polarization current term was introduced into the equations to link the frequency dependence of  $\varepsilon$  into the algorithm. Thus, the performance of the FDTD calculations do not suffer severely from the inclusion of dispersive media. For convenience, we normalize frequencies by  $\omega_{ab}$  since the FDTD calculations are performed with a unit length that is normalized by the wavelength of interest.

### Microresonator structure

To study the impact of a dynamically changing EIT medium inside of an optical resonator, we used our EIT FDTD code to perform two-dimensional calculations on the corrugated waveguide microcavity system shown in Figure 3. Previous studies found that the introduction of a highly dispersive medium in the cavity of this system had the effect of increasing the  $Q$ -factor while decreasing the transmission line-width of



**Fig. 3:** Geometry of the microcavity waveguide system used for calculating the effect of a dynamically changing EIT medium inside of an optical resonator. The shaded region indicates the area that contained the EIT medium.

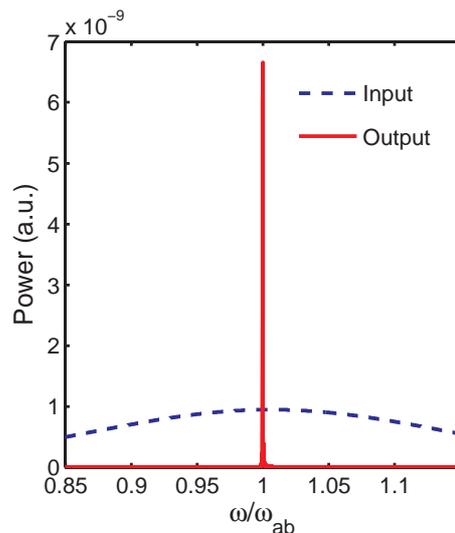
the waveguide [5]. However in those calculations, the EIT region was under a static Rabi frequency configuration, e.g. the permittivity of the EIT did not change as a function of time.

As shown in Figure 3, a silicon waveguide was defined in an air background in the center of the computational domain. A periodic arrangement of three air holes, lattice spacing  $a$  and radius  $0.35a$ , were positioned on each side of an EIT region in the waveguide to form a resonator cavity. The holes act as partially transmitting Bragg mirrors, and the cavity was  $1.4a$  in width. The source pulse was TE-polarized (E-field within the plane of the paper) and propagated from left to right starting from the black vertical line on the far left of the waveguide. Detectors were located at the vertical lines on either side of the microcavity as indicated. Only the material in the cavity of the waveguide was an EIT medium, as indicated by the darker color in Figure 3, with a permittivity given by Equation (2). The computational domain is terminated on all sides by a perfectly-matched-layer absorbing boundary condition to avoid reflections from the domain boundaries. The EIT material was designed such that the  $ab$ -transition frequency coincided with the resonance frequency of the cavity.

At the beginning of the calculation, the control field Rabi frequency  $\Omega_c/\omega_{ab}$  was set to 0.1612. The permittivity of the EIT in this state is given by the solid line in Figure 2. Then, after the resonance of the cavity was allowed to establish,  $\Omega_c/\omega_{ab}$  was linearly decreased to 0.0 (the ‘off’ state which is given by the dashed line in Figure 2) over the period of 1000 time steps. The Rabi frequency was left in the off state for the remainder of the calculation.

### Results

Figure 4 shows the power spectrum of the input pulse (dashed line) and of the resonator cavity (solid line) after the control field is turned off. As shown, the power increased almost an order of magnitude due to the dynamic tuning of  $\Omega_c$ . The total power in the cavity was calculated from the data collected by the two detectors and was found to be slightly less than that of the input pulse. Thus, the increase of power at the  $\omega_{ab}$  is not in violation of the conservation of energy. Physically, one can understand this as a squeezing of the energy



**Fig. 4:** Power spectrum of the input pulse (dashed) and of the cavity (solid) after the control field was turned off.

of the cavity resonance into the single frequency of the  $ab$ -transition by the dynamic change in the steep dispersion of the EIT. Since we have assumed a zero decoherence rate, the dominating limitation of this system is the radiation of the cavity to the surroundings above and below the cavity region.

### Conclusion

We modelled a microcavity resonator system which possesses a dynamic, highly dispersive medium (such as an EIT medium) inside of the resonator cavity. We showed that the dynamic tuning of such a microresonator cavity shows a response where the power of the cavity mode is channelled into a single frequency, increasing the power output and the lifetime of the cavity dramatically.

### Acknowledgments

Funding for this work was through the Swedish Foundation for Strategic Research (SSF) under the INGVAR program, the SSF Strategic Research Center in Photonics, and the Swedish Research Council (VR).

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