

# Excitation of Leaky modes in the System of Coupled Waveguides

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**Abstract:** In this paper we study a system of coupled waveguides where the number of guided modes  $M$  is less than number of individual waveguides  $N$ . We study in details leaky modes in this system both theoretically and experimentally. In particular we show that leaky modes are excited from the side of the waveguide system at the same time when the reflection coefficient for the exciting beam goes to zero.

## Introduction

Uniform system of coupled waveguides is a group of equidistantly placed identical waveguides. It could be a multilayer dielectric mirror or a group of channel waveguides in the dielectric layer on top of lower index substrate. Recently this kind of waveguide systems attracted attention due to possibility of creation of high-power lasers [1]. Earlier we studied the process of light propagation in a uniform system of coupled waveguides [2-3]. It was noted that the Bragg diffraction process in this system is of particular interest [4]. In this paper we are trying to examine the Bragg diffraction process in details for both edge and side incidence of the exciting light on the system of coupled waveguides.

## Theory

It was shown earlier [2] that number of modes in a system of coupled waveguides depends on number of waveguides  $N$  and the refractive indices of guides and gaps. In this paper we study the system with the following parameters: number of waveguides  $N=50$ ; waveguide layers  $n_f=1.465$ ,  $h=1.1\text{mkm}$ ; gaps  $n_s=1.46$ ,  $s=1.3$ ; cover and substrate also have refractive index 1.46 [6]. In the case of weak waveguides number of guided modes  $M$  is less than number of waveguides  $N$ . For this particular system we have  $M=34$  and the

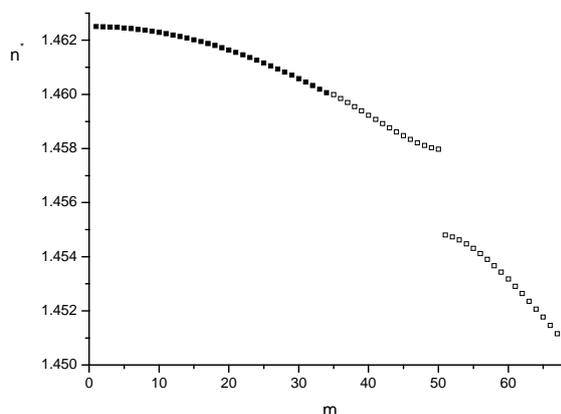


Fig. 1: Effective refractive indices of modes

rest of the modes are leaky. Figure 1 presents the dependence of the effective refractive indices of modes  $n^*$  on the mode order  $m$ . Leaky modes exhibit light radiation into the adjacent media during the propagation along the system. Angle of propagation through the side surfaces of the system is given by following equation:

$$\cos \theta = \frac{n^*}{n_s} \quad (1)$$

Figure 2 presents the dependence of this angle on the mode order for modes  $m = 35 \dots 50$  (curve 1) [5].

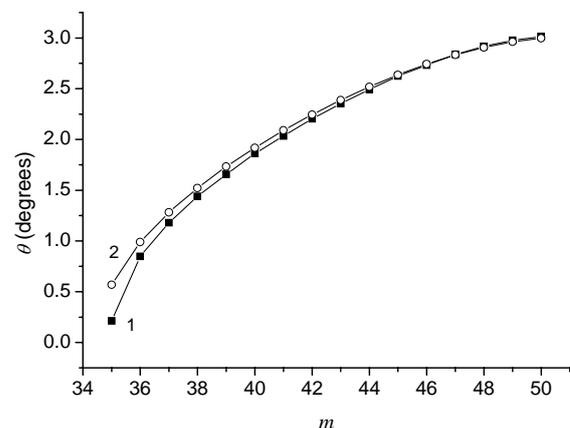


Fig. 2: Leak angles for different modes

Leaky modes in the system can be excited by the light beam incident on the side surface of the system. As our system is periodic it exhibits well known Bragg reflection. First we calculated the dependence of the reflection coefficient for side incident beam. Figure 3 presents these results.

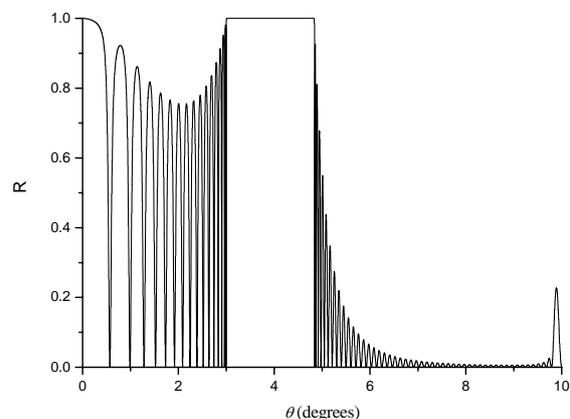


Fig. 3: Reflection coefficient for side-incident beam

One should note that in the range of incidence angles from 0 to 3° there are peaks and dips and the number of dips is equal to the number of leaky modes of the waveguide system in the first permitted band:  $N - M = 16$ . Then we compared the positions of these minima (fig.2 curve 2) with the dependence of leak angle for modes calculated earlier and that they are in good agreement. These two things led us to conclusion that leaky modes are excited when the angle of incidence coincides with the minimum of the reflection curve  $R(\theta)$ .

Treating our waveguide system as a photonic crystal it is convenient to present a field inside the system as a combination of Floquet-Bloch waves with the Bloch wavevector  $K_b$  given by the following formulas [7]:

$$\begin{aligned} \cos K_b \Lambda &= \cos \kappa_f h \cdot \cos \kappa_s s - \\ &\frac{1}{2} \left( \frac{\kappa_s}{\kappa_f} + \frac{\kappa_f}{\kappa_s} \right) \cdot \sin \kappa_f h \cdot \sin \kappa_s s \end{aligned} \quad (2)$$

where  $\kappa_f = k\sqrt{n_f^2 - n^{*2}}$ ,  $\kappa_s = k\sqrt{n_s^2 - n^{*2}}$  are transverse components of plane wave wavevectors in guiding and intermediate layers respectively,  $k = 2\pi/\lambda$ ,  $\lambda$  is wavelength of light on free space. According to [7] the reflection coefficient  $R(\theta)$  can be calculated as follows:

$$R(\theta) = \frac{|C|^2}{|C|^2 + (\sin K_b \Lambda / \sin NK_b \Lambda)^2} \quad (3)$$

and is mainly determined by rapidly changing term  $\sin K_b \Lambda / \sin NK_b \Lambda$ . Coefficient  $|C|^2$  is a structure factor and is given by

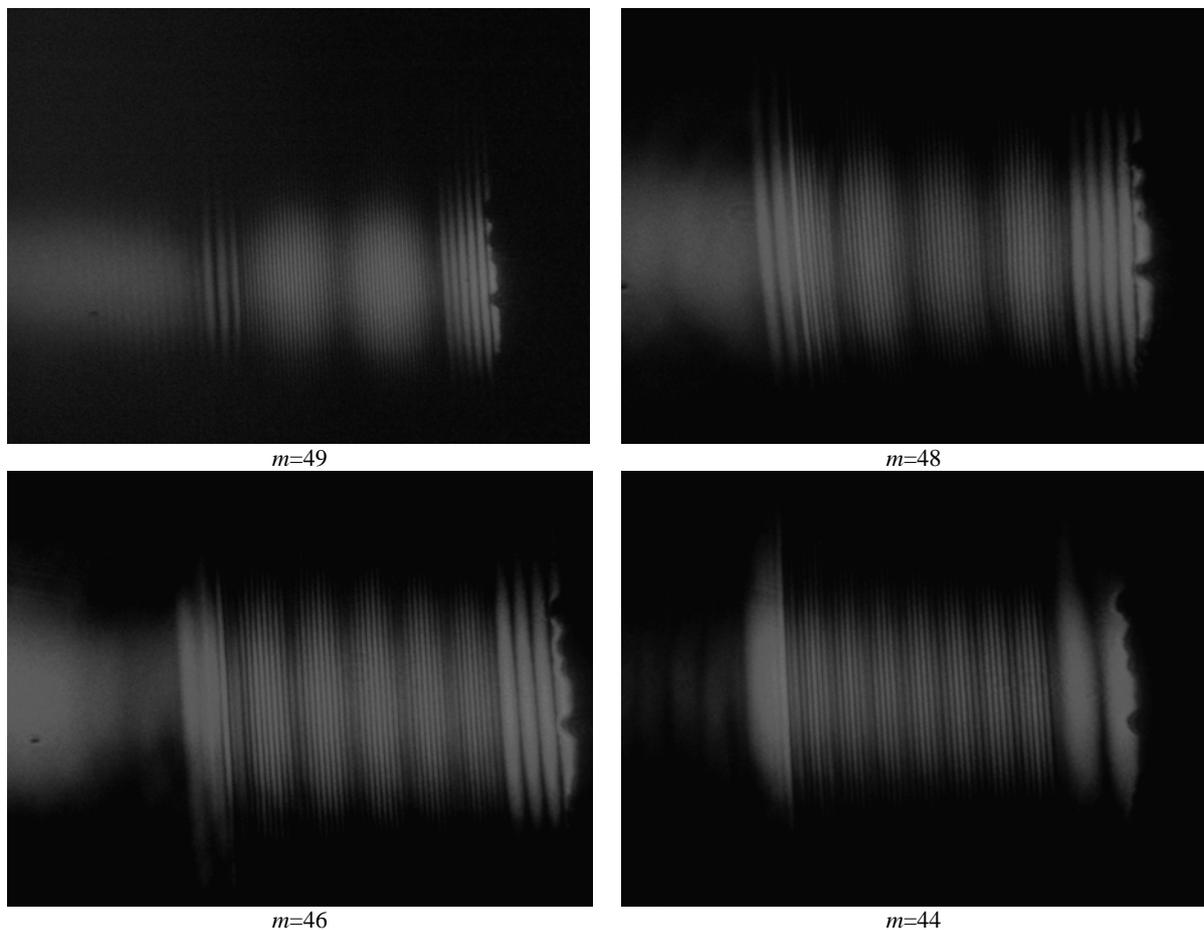
$$|C|^2 = \frac{1}{4} \left( \frac{\kappa_f}{\kappa_s} - \frac{\kappa_s}{\kappa_f} \right)^2 \cdot \sin^2 \kappa_f h \quad (4)$$

It slowly changes with the variation of incidence angle. In the case of non-zero  $|C|^2$  the condition

$$\frac{\sin NK_b \Lambda}{\sin K_b \Lambda} = 0 \quad (5)$$

is actually the dispersion relation for leaky modes of the system.

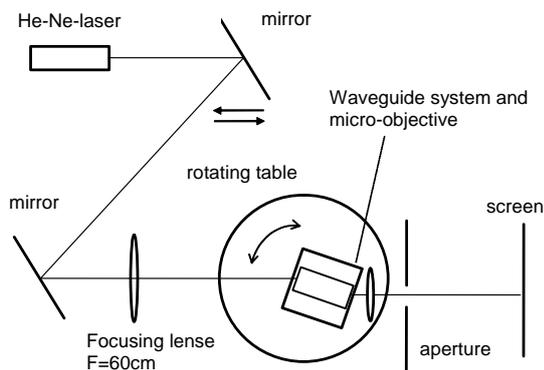
We calculated the field inside the system when it is illuminated by side-incident beam. It was found that the minima of reflection correspond to the maxima of energy accumulation in the system that means that leaky modes are excited. Distribution of field in that case corresponds to the calculated earlier mode profile of leaky modes. For angles corresponding to the maxima of the reflection energy accumulation is practically absent. This behaviour is analogous to widely used Fabry-Perot interferometer where the energy is also accumulated in the minima of transmission.



**Fig. 4:** Leaky modes field at the output edge of the waveguide system

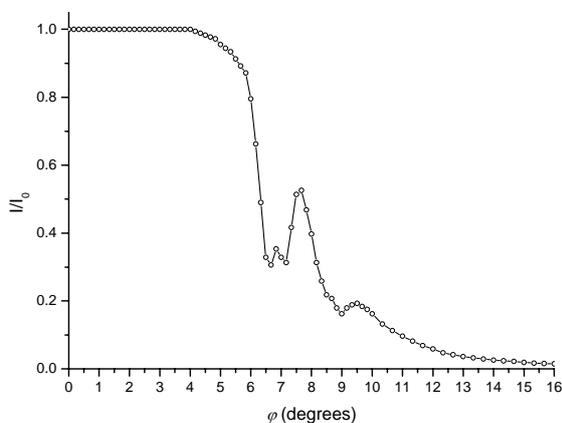
## Experiment

We observed the excitation of leaky modes with the side-incident beam first in [8]. In this paper we want to study this phenomenon in more details. We used the setup shown in figure 5. Transmitted and reflected beams were observed on the screen put at 2.5m distance. For observation of the intensity distribution at the output edge of the system its image was created on the screen using micro-objective with a magnification factor  $M=300$ . Light beam falls onto the edge of the sample in the buffer layer area, refracts on the air-substrate interface and then drop onto the side surface of the waveguide system (see fig.7).



**Fig 5:** Experimental setup

For angles  $3 < \theta < 4.8^\circ$  corresponding to the photon forbidden band of the system we observed bright reflection spot and the intensity of the transmitted wave was practically zero. For angles below  $3^\circ$  we observed several dark stripes in the reflected beam and the matching bright stripes in the transmitted beam. Each of these stripes corresponds to one of leaky modes propagating in the waveguide system. For modes close to the forbidden band angular distance between modes was about  $0.1^\circ$  that is several times less that the angular aperture of the exciting beam incident on the side surface of the system that



**Fig. 6:** Waveguide system transmission for edge excitation

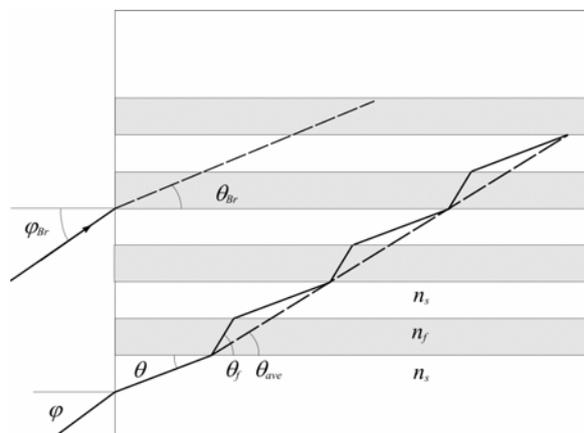
was about  $0.6^\circ$  for the beam having  $50 \mu\text{m}$  diameter. That is why we observed simultaneous excitation of several (3-4) leaky modes. For angles in the middle of the allowed band ( $\theta \sim 1-2^\circ$ ) we observed excitation of 1-2 modes as the distance between modes is larger. We also observed the same behaviour for angles in the second allowed band  $4.8 < \theta < 9.9^\circ$ .

Figure 4 presents images of output edge of the system excited by a  $230\mu\text{m}$  beam incident on its side. Shown photographs correspond to the 2<sup>nd</sup>, 3<sup>rd</sup>, 5<sup>th</sup> and 7<sup>th</sup> dips in the reflection counting from the forbidden band. This means that they are present the 49<sup>th</sup>, 48<sup>th</sup>, 46<sup>th</sup> and 44<sup>th</sup> modes respectively. The 50<sup>th</sup> mode that is closest to the forbidden band has the lowest losses and the narrowest resonance. It is difficult to excite it efficiently that is why we do now show its image. One can see that that envelope of the mode fields has periodicity depending on the detuning from the band edge that is in full coincidence with the calculation results.

For incidence angles inside the photonic forbidden band  $3 < \theta < 4.8^\circ$ , modes of waveguide system, can not be excited by a side-incident beam. However the edge-incident beam under the Bragg angle

$$\varphi_{Br} = \arcsin \frac{\lambda}{2\Lambda} \quad (6)$$

excites both low-loss Bragg modes [6] that can be seen at figure 6. This figure presents the power that was registered at the output edge of the waveguide system (collecting lens and the photodetector were used in this experiment instead of screen). It is important that both Bragg modes are excited simultaneously and are leaky modes in our case. The observed peak clearly shows that the Bragg modes have the lowest losses among other leaky modes and thus could potentially be used for mode selection in lasers. We should note one interesting fact (see Figure 7). If we use the refraction law and recalculate the edges of forbidden band into angles in air ( $4.38^\circ-7.02^\circ$ ) then we found that the Bragg angle  $\varphi_{Br} = 7.58^\circ$  lies outside the range calculated from the reflection



**Fig.7:** Scheme explaining the geometry of propagation in the multilayer structure

curve  $R(\theta)$ . At first sight it contradicts to the common sense, but there is very simple explanation. Angle of light propagation in the guiding layers  $\theta_f$  differs significantly from  $\theta$  despite the low refractive index difference ( $n_s \cos \theta = n_f \cos \theta_f$ ). Average angle of light propagation through the system can be determined from simple geometrical consideration and appears to be in  $3.81^\circ$ - $5.53^\circ$  range ( $(h+s) \text{ctg } \theta_{ave} = h \text{ctg } \theta_f + s \text{ctg } \theta$ ). If we recalculate the Bragg angle into the waveguide system using the averaged refractive index we get  $5.17^\circ$ . Thus there is no contradiction and the Bragg angle lies inside the forbidden band as it should. One should note that the averaged refractive index calculated from the formula  $n_{ave} \cos \theta_{ave} = n_s \cos \theta$  depends not only the layers parameters but also on the incidence angle.

### Conclusions

Thus our study of Bragg diffraction in the system of coupled waveguides and examination of light reflection from the side surface of the system complement each other and as a whole give us the total understanding of the processes taking place in the system. In particular for the system having number of guided modes  $M$  smaller than the number of waveguides  $N$  it was found that leaky modes can be excited by a side-incident beam and there are  $N-M$  of them. It was shown that leaky modes are excited from the side of the waveguide system at the same time when the reflection coefficient for the exciting beam goes to zero. It was verified experimentally that the Bragg modes have the lowest losses among leaky modes that can be used for mode selection in lasers

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