Photonic crystals in diamond: Cavity Q – Mode volume influence on the design

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Abstract: We present a qualitative analysis of the influence of the mode volume on a planar photonic crystal (PC) cavity quality factor (Q) in diamond. Optimal waveguide based cavity geometry with minimized vertical losses is considered. The mode width and slab thickness are varied to provide maximum Q. The results are compared to the similar designs in silicon. The optimal geometric parameters for diamond cavities are derived. These results are supported by 3D Finite Difference Time Domain (FDTD) calculations on double heterostructures (DHs) cavities. The highest $Q\approx 135,000$ is demonstrated.

Introduction

2D Photonic Crystal Structures in diamond are being considered as an attractive architecture for the control and manipulation of atom-photon dressed states [1]. Several designs for the high-Q cavities on diamond were recently reported [2, 3], while the best result of $Q\approx 70,000$ is obtained for the DH cavity design [3]. This result is more than one order of magnitude lower than similar designs in silicon [4, 5]. This major discrepancy rises the question of the physical mechanism responsible for the dielectric constant influence on photonic crystal cavity Q. Here, we discuss the difference between quality factors in high-ε (silicon) and low-ε (diamond) slab PC cavity designs.

The considered PC cavities consists of a membrane suspended in air and periodically modulated by a triangular array of air holes. The design resonances are TE-like modes located at the 1st bandgap.

Qualitative approach

The cavity mode is confined laterally by the Bragg reflection and vertically by total internal reflection. Therefore, the quality factor (Q) can be divided into two major components, for the lateral (Q_l) and the vertical (Q_v) losses [5]. Since Q_l is determined by the number of PC periods around the cavity, by embedding the cavity in a sufficiently large PC, the value of Q_l can be made arbitrarily large. Therefore, $Q=Q_v$ is assumed.

We discuss the cavities formed as spatial modulation of a PC waveguide, providing mode localization, that demonstrated the best Q values in both silicon [4, 5] and diamond [3]. The PC waveguide is defined by a linear defect into the PC slab in the ΓJ direction, with one filled row of holes (see Fig. 1). The cavity design is based on the lowest waveguide mode.

The vertical power losses (P_v) in a cavity are proportional to the integral of the Fourier Transform (FT) squared - $|FT(H_y)|^2$ (H_y - magnetic field in y direction) in the light cone area, i.e. in $k_l<k_0$, where $k_l=(k_x,k_z)$ and $k_0=2\pi/\lambda_0$. Following the inverse problem approach [5], the cavity mode is represented as a product of waveguide field and a slowly varying envelope function. In order to minimize P_v an ideal Gaussian envelope function is assumed [5]:

$$FT_2(H_y)=\sum_{k_{x_0},k_{z_0}} sign(k_{x_0})\exp\left(-\left((k_x-k_{x_0})\frac{\sigma_x}{\sqrt{2}}\right)^2\right)\exp\left(-\left((k_z-k_{z_0})\frac{\sigma_z}{\sqrt{2}}\right)^2\right) \quad (1)$$

where: σ_x and σ_z are the modal widths in real space in x and z directions respectively, and ($\pm k_{x0} \pm k_{z0}$) are the J points.

We assume that cavity resonance is close to the waveguide one, i.e. $\omega_0\approx\omega_w$. Since silicon dielectric constant is higher than that of diamond, the PC

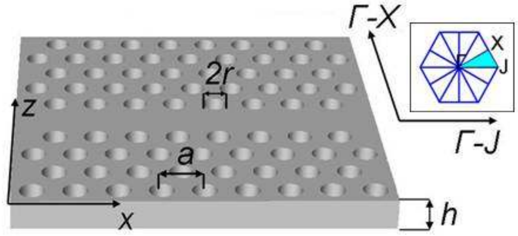


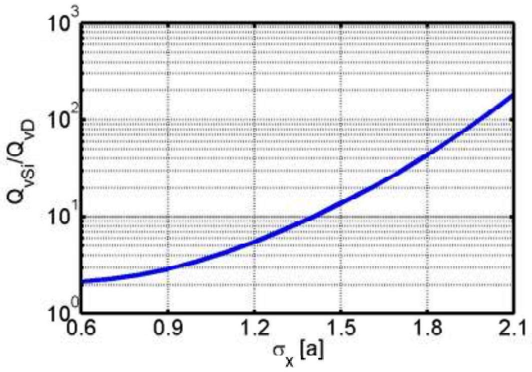
Fig. 1: The notation of PC waveguide. In the inset the 1st Brillouin zone of the triangular lattice with high symmetry points are depicted.

waveguide mode frequency in silicon is lower than that of diamond ($\omega_{wSi}<\omega_{wD}$). Consequently, cavity mode frequencies satisfy $\omega_{0Si}<\omega_{0D}$. This difference in the resonant frequency is responsible for the different light cones, thus resulting in exponentially different Q_v in silicon and diamond [3].

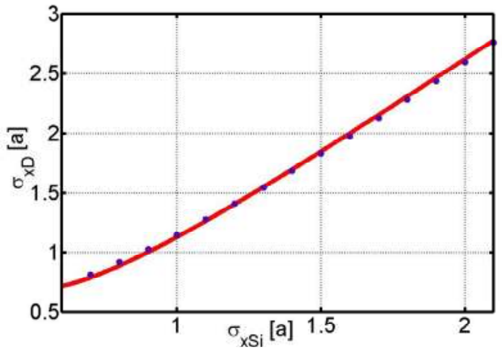
In Fig. 2(a) the ratio between the vertical quality factors (Q_{vSi}/Q_{vD}) as a function of modal width in x direction (σ_x) for the ideal envelope cavities (Eq. 1) is depicted. Note, that $\sigma_{xD}=\sigma_{xSi}$ is assumed. The mode widths in the z direction are defined by the bandgap confinement, and obtained from the fit to DHs in [3] and [4]. From the figure Q_{vSi}/Q_{vD} exponentially increases with σ_x, since the vertical losses (P_v) in silicon and diamond light cones are exponentially different.

The exponential dependence of Q_v on σ_x suggests that higher P_v in diamond can be compensated by an increased σ_{xD}, thus improving $|FT(H_y)|^2$ localization. In Fig. 2(b) σ_{xD}, vs. σ_{xSi} values leading to $Q_{vSi}=Q_{vD}$

are plotted. There is an accurate fit to σ_{xD}: $\sigma_{xD}=[A+B(\sigma_{xSi}-C)^2]^{1/2}$, while for $\sigma_{xSi}>0.9a$, it may be approximated by a linear relation. A general expression for $\sigma_x(\varepsilon_1)$ vs. $\sigma_x(\varepsilon_2)$, where ε_1, ε_2 are respective slab dielectric constants, will be presented.



(a)



(b)

Fig. 2: (a) Q_{vSi}/Q_{vD} vs. σ_x. (b) σ_{xD} (that provides $Q_{vSi}/Q_{vD}=1$) vs. σ_{xSi} . In blue σ_{xD} is shown. In red the fit to $\sigma_{xD}=[A+B(\sigma_{xSi}-C)^2]^{1/2}$.

In the following the influence of slab thickness (h) on the vertical quality factor (Q_v) is discussed. Although the analysis given here treats the diamond cavities with a Gaussian envelope, it can be extended to other materials and to any PC waveguide based design, including DH cavities. Here, we assume that the cavity resonance frequency is close to the waveguide one, i.e. $\omega_0\approx \omega_w$. As one can observe from Fig. 3(a), the mode's k decreases with an increase in the slab height. This k decrease in the light cone radius results in a subsequent decrease of vertical power losses, thus significantly improving the Q_v.

In Fig. 3(b, c) we present the Q_v behavior in two diamond Gaussian envelope cavities with different mode widths in x direction (σ_x) as a function of the slab thickness. We can see that the increase in the slab thickness from $0.6a$ to $0.9a$ increases the Q_v by the factor that varies from 3.45 for the $\sigma_x=1.5a$ to 13 for $\sigma_x=2.6a$. Note that slab thickness increase is bounded by a subsequent decrease in the bandgap, which sustains lateral confinement of cavity mode. Therefore, slab thickness increase is limited by the photonic bandgap existence condition.

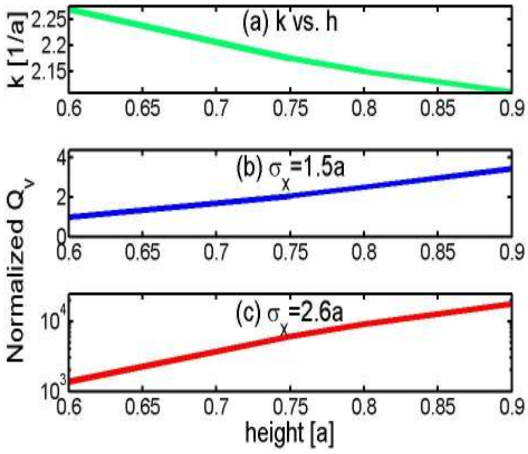
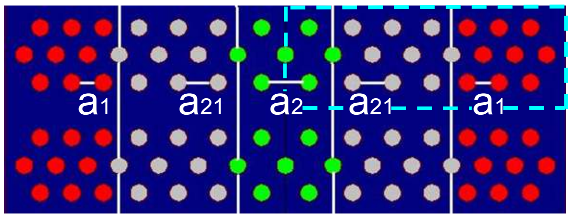


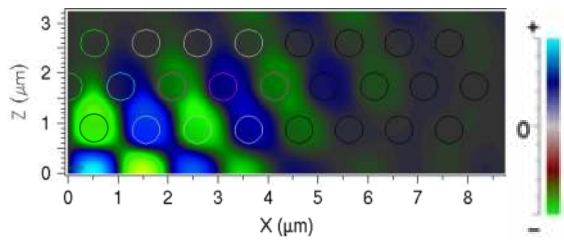
Fig. 3: Slab height influence on Q_v. (a) Diamond light cone radius k vs. h in waveguide based cavities. (b) Q_v vs. h for $\sigma_x=1.5a$. (c) Q_v vs. h for $\sigma_x=2.6a$. All quality factors are normalized to the cavity with $h=0.6a$ and $\sigma_x=1.6a$.

DH Analysis

These qualitative results are checked for the diamond modified double heterostructure (DH)] cavities [4] with 3D FDTD calculations. In this structure the confinement in x direction is formed by two different layers of elongated photonic crystals: a_1, a_{21} ($a_2>a_{21}>a_1$), while the confinement in z (lateral) direction is supported by the band gap (see Fig. 3(a)). When the number of a_{21} periods is increased from 0 to 3, σ_{xD} increases from $1.93a$ to $2.08a$, resulting in Q_v increase from $67{,}000$ to $135{,}000$. Although DH is not an ideal Gaussian, still Q_v vs. σ_x is nearly exponential. The H_y profile is shown in Fig. 3(b). Note that the increase in slab thickness in DH



(a)



(b)

Fig. 3: (a) Modified DH with $3a_{21}$ periods. (b) The H_y profile in the 1st quarter of the DH at $y=0$ plane.

increases Q_v from 37,000 to 67,000 [3]. This can be easily explained by the decrease in the vertical power losses. In a DH cavity, the resonant mode frequency is found between the 1st waveguide mode and the dielectric band edge, while the dielectric band edge dependence on a slab thickness is similar to that of the waveguide one. Therefore, similar decrease in the light cone radius (k) as the slab thickness increases, is expected.

Conclusions

We studied how the modal width and slab thickness increase can significantly improve the cavity Q. Moreover, for the QED application, since Q dependence on σ_x is exponential, while mode volume V dependence is linear, it is worth to compensate high vertical losses with increased mode width. The slab thickness increase in the waveguide based cavity designs improves the Q_v by decreasing the light cone. In the diamond cavity studied here an optimal cavity thickness of $0.9a$ is obtained. Similar behavior is observed at diamond modified DH cavities, where the increase in σ_x results in $Q_v=135{,}000$.

Acknowledgments

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