

Quantum State Tomography in Static Optical Circuits

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Full characterization of quantum states is of increasing importance as ever more complex quantum systems are realized. Quantum state tomography of these systems is a difficult task because it requires a large number of different measurements to be taken. This is because measurement results in quantum mechanics are probabilistic, so many measurements must be taken of near identical copies of the system determine quantum mechanical expectation values. Furthermore, many different expectation values must be measured to fully characterize the density matrix because it occupies a high dimensional space that cannot be fully determined by projection onto a single measurement basis. Thus typical approaches to quantum state tomography require reconfigurable optical setups to measure expectation values in different bases [1].

We show that it is in fact possible to achieve quantum tomography using a static measurement setup without any reconfigurable elements [2]. Our approach is to map the density matrix to a sparse state using a single linear transformation, then exploit the introduced sparsity to recover the density matrix from just one set of measured expectation values. Sparsity is introduced to the state by using a linear transformation that maps N input modes to M output modes where $N < M$, as illustrated in Fig. 1. Given the exact form of the transformation is carefully chosen, this method can allow any input density matrix, $\hat{\rho}_{\text{in}}$, to be uniquely determined just by measuring the expectation values of $\hat{\rho}_{\text{out}}$ in a single measurement basis.

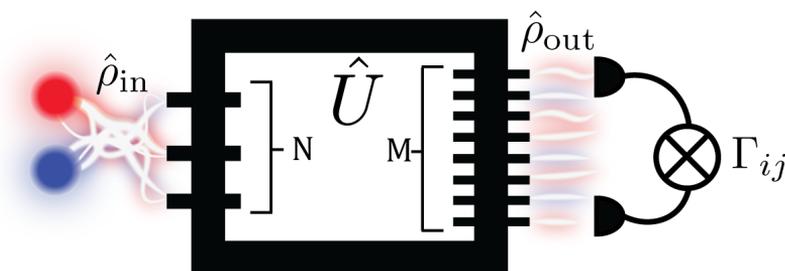


Fig. 31. Linear transformation on unknown input mixed two photon state that maps it to a sparse output state.

The ability to perform quantum state tomography in a single measurement basis is significant because it shows that a static measurement apparatus is sufficient for quantum state tomography. This means the need to reconfigure measurement setups is avoided, significantly simplifying the quantum tomography procedure. This could reduce noise in characterization of quantum systems, as well as opening up a path for characterization of more complex systems, where reconfiguring the measurement apparatus is very difficult or time consuming.

To demonstrate this approach we consider tomography of a spatially entangled two-photon mixed state within a photonic chip. Generally characterization of such a state would be achieved using a reconfigurable measurement setup [3], however we show that full tomography is possible with just a static optical circuit. We aim to determine

the density function of an input state, with entanglement between two input waveguides characterized by a density matrix such as that shown in Fig 2(a). We design an integrated optical circuit shown in Fig. 2(b), which performs a specially optimized linear transformation to spread the unknown state from two input to four output waveguides. This circuit implements a hybrid of discrete and continuous quantum walks on the input photon state, first splitting the two input modes into four, then these four modes are coupled together forming a coupled waveguide array, where photons can perform a continuous quantum walk across the waveguides.

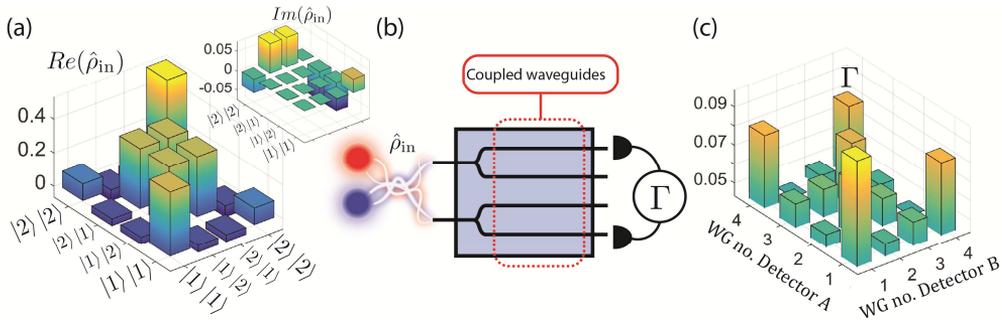


Fig. 2. (a) Example density matrix of a two-photon and two mode mixed state. (b) Linear optical circuit mapping the two mode state to a sparse four mode state. (c) Two photon spatial correlations of the state after the linear transformation, which allow tomographic reconstruction of the input state.

After the hybrid quantum walk, the photons are detected using single photon detectors, and correlations in the arrival times at the detectors give the expectation values of finding one photon in waveguide i and the other in waveguide j . This gives the two-photon spatial correlation distribution shown in Fig. 2(c). Due to the specially chosen form of the linear optical circuit, these measured correlations are sufficient to uniquely determine the full form of $\hat{\rho}_{\text{out}}$. Since $\hat{\rho}_{\text{out}}$ and $\hat{\rho}_{\text{in}}$ are linked by a simple linear transformation, $\hat{\rho}_{\text{in}}$ is also uniquely determined. Thus the input state, $\hat{\rho}_{\text{in}}$, in Fig. 2(a) can be reconstructed from the measured correlations in Fig. 2(c), so full quantum state tomography is achieved using the static optical circuit. Furthermore, we develop an optimal design which is robust in presence of device imperfections or measurements errors.

The key advantages of our approach is the robustness and experimental simplicity of the device and measurement process. This could facilitate the integration of cryogenically cooled on-chip single-photon detectors, allowing fully on-chip quantum state tomography.

References

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