Finite Element Analysis of Optical Notch Filters based on Photonic Crystals

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A Photonic crystal based notch filter is analyzed by means of Finite Element Method. Simulation results show that the filter exhibits a notch with very narrow bandwidth and its position can be easily tuned with high accuracy over a wide range of wavelengths.

Keywords: Notch Filter, Photonic Crystals, Finite Element Method

Introduction

Photonic crystal (PCs) based devices are expected to meet the demand of the modern optical systems requiring compact devices able to carry out more and more complicated functionalities. It has already been shown low-loss and very compact splitters, sharp bend waveguides, switches, lasers and filters [1]-[4] can be obtained using PCs. In particular, several typology of optical filters can be obtained by exploiting the strong wavelength dependence of the PCs characteristics [1]-[3]. Add-drop filters have been proposed or by coupling two waveguides by means of a resonant cavity [1], [2], or by considering a PCs directional coupler [3]. In this work notch filters based on a PCs waveguide coupled with a resonant cavity are numerically analyzed by means of finite element method (FEM) approaches both in time and in frequency domain [3]. The two approaches present complementary properties. The first one allows a wide band analysis (hundreds of nm) with just one simulation, whereas the latter is very useful when the impact of the material dispersion effects can not be neglected and/or a limited range of frequency must be considered. The FEM is a very powerful method to analyze PCs based devices. It is able to describe with high accuracy any kind of geometrical shape and refractive index profile without heavy computational resources. These properties have been already successfully used to investigate the impact of the geometrical variations on the spectral properties [5]. The analysis reported in this work shows that compact optical filters can be obtained having bandwidth compatible with the channel spacing of the present wavelength division multiplexing (WDM) systems. This result can be achieved by working both on the coupling coefficient between the waveguide and the cavity and on the cavity quality factor Q. In addition, by changing the characteristics of the PCs cells between the waveguide and the resonant cavity, an accurate tuning of the spectral characteristics over a wide range is possible.

Numerical methods

The analysis of PCs based devices can be done by means of numerical approaches able to handle reflected and scattered waves. This is possible both in the time domain and in the frequency domain.

Consider a two-dimensional photonic crystal waveguide in the yz-plane, with no variation in the x direction, and anisotropic media described by relative permittivity and relative permeability diagonal tensors $\overline{\overline{\varepsilon}}_r$ and $\overline{\overline{\mu}}_r$ respectively. In this condition, the analysis of the electromagnetic field propagation can be performed simply by considering the x-component Φ_x of the electric or of the magnetic field, in the case of TE or TM waves respectively.

Time Domain Approach

By assuming Φ_x of the form

$$\Phi_x(y,z,t) = \phi_x(y,z,t)\, e^{j\omega t}, \tag{1}$$

being ω the angular frequency and ϕ_x the complex amplitude, the wave equation reads:

$$\frac{q_x}{c^2}\frac{\partial^2 \phi_x}{\partial t^2} + 2j\omega\frac{q_x}{c^2}\frac{\partial \phi_x}{\partial t} - \omega^2\frac{q_x}{c^2}\phi_x \; - \frac{\partial}{\partial y}\left(p_z\frac{\partial \phi_x}{\partial y}\right) - \frac{\partial}{\partial z}\left(p_y\frac{\partial \phi_x}{\partial z}\right) = 0, \tag{2}$$

with p_i and q_i, $i = x, y, z$, the elements of the tensors $\overline{\overline{p}}$ and $\overline{\overline{q}}$. In particular $\overline{\overline{p}} = \overline{\overline{\mu}}_r^{-1}$, $\overline{\overline{q}} = \overline{\overline{\varepsilon}}_r$when $\phi_x = E_x$ for TE modes, $\overline{\overline{p}} = \overline{\overline{\varepsilon}}_r^{-1}$, $\overline{\overline{q}} = \overline{\overline{\mu}}_r$ when $\phi_x = H_x$ for TM modes. Choice (1) allows the propagation analysis of the amplitude ϕ_x rather than of the original field Φ_x which varies very quickly in time. Larger time step can thus be considered for the simulations, thus reducing the computational time required for a given time interval analysis. By applying the standard finite element technique to the spatial coordinates of (2) and to the time variable as well, it yields the following recursive algebraic formula:

$$[P]\{\phi_x\}_{n+1} = [Q]\{\phi_x\}_n - [P]^*\{\phi_x\}_{n-1}$$

with

$$[P] = \frac{[B]}{c^2\Delta t} + \frac{j\omega}{c^2}[B] + \frac{\Delta t}{2}([A] + [C] - \frac{\omega^2}{c^2}[B]), \;\; [Q] = \frac{2}{\Delta t c^2}[B].$$

The elements of the $[A]$, $[B]$ and $[C]$ matrices are defined by the proper products of the shape functions and their derivatives [6]. It is important to highlight that the described approach does not neglect the second order derivatives in t. This allows the propagation analysis of very narrow time pulses. By Fourier transforming the temporal evolution of the field, spectral information over a wide band can be obtained with only one simulation. Notice that material dispersion effects should be considered through convolution integral. Nevertheless, as the considered devices are very short, they can be neglected without affecting substantially the result accuracy.

Frequency Approach

Starting from the curl-curl equation in the frequency domain and considering Φ_x and the medium as above, it yields:

$$\frac{\partial}{\partial y}\left(p_z\frac{\partial \Phi_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(p_y\frac{\partial \Phi_x}{\partial z}\right) - \frac{\omega^2}{c^2}q_x\Phi_x = 0. \tag{3}$$

The application of the standard finite element technique to the spatial coordinates of (3) directly yields

$$\left([A] + [C] - \frac{\omega^2}{c^2}[B]\right)\{\Phi_x\} = [D]\{\psi_x\} \tag{4}$$

where $\{\psi_x\}$ is the known vector of the input field ψ_x; it is defined on a portion of the domain boundary Γ. Once chosen the input field ψ_x and the working frequency, the resolution of the standard algebraic equation (4) provides the field distribution on the whole domain. Perfectly Matched Anisotropic layers (PMA) have been used to enclose the domain in both approaches [7].

Results

The analyzed structure, consists on a square lattice of circular dielectric rods in air, with a $85.7\,nm$ radius, having a lattice constant of $600\,nm$. It is reported in figure 1. The coupling among the waveguide mode and those of the resonant cavity causes the filtering effect according to the coupling coefficient and to the Q factor [1]. Higher the Q factor, more selective is the spectral response. To increase Q, the number of the rods separating the cavity and the waveguide must increase [8]. Unfortunately, this also reduce the coupling; if the rod number is too high, the waveguide and the cavity are uncoupled. In the PCs under investigation, a trade-off is introduced by separating the cavity and the waveguide with two rods; it is shown in figure 1. The spectral response has been firstly calculated by means of the time domain approach to obtain information over a wide range of wavelength with only one simulation. The obtained curve is shown in figure 1. Very narrow notches are evident near $1.51\,\mu m$ and near $1.7\,\mu m$. For this kind of spectral response, a time domain approach is not the best tool. In the high Q value cavity, the decay rate of the electromagnetic energy stored in the cavity is very low and this forces very long time simulation in order to obtain the necessary frequency resolution. On the contrary, by means of the frequency approach the spectral characteristics near the notch can be evaluated with the user desired frequency resolution. The trasmittance spectrum near the notch at $1.51\,\mu m$ is reported in the inset of figure 1. It was obtained by means of the frequency approach. The full width at half maximum (FWHM) of this filter is less than $0.7\,nm$ and the center band transmittance is about $-27\,dB$.

The notch position can be tuned over a wide range of wavelengths by changing the characteristics of the crystal cell between the waveguide and the cavity. In figure 2(a) the transmittance spectra obtained by a change of the 5% in the refractive index of the rod near the cavity or of the rod near the waveguide or both are reported. This figure points out an important property of this filter; by changing the rod near the cavity, strong variations are obtained, whereas by changing that one near the waveguide a more accurate tuning is possible. In all cases the transmittance value at center band is almost unchanged. The same behaviour is observed by changing the rod radius. In figure 2(b), notch shift obtained by changing the radius of the rod near the cavity or the one near the waveguide is reported. The rod near the waveguide allows to control the notch position on a scale comparable with the channel spacing, while the rod near the cavity allows to control the notch position over a range of several nanometers.

In conclusion, it has been shown that highly selective notch filters accurately tunable within a range of several nanometers, can be obtained by means of a proper design of the PC structures.

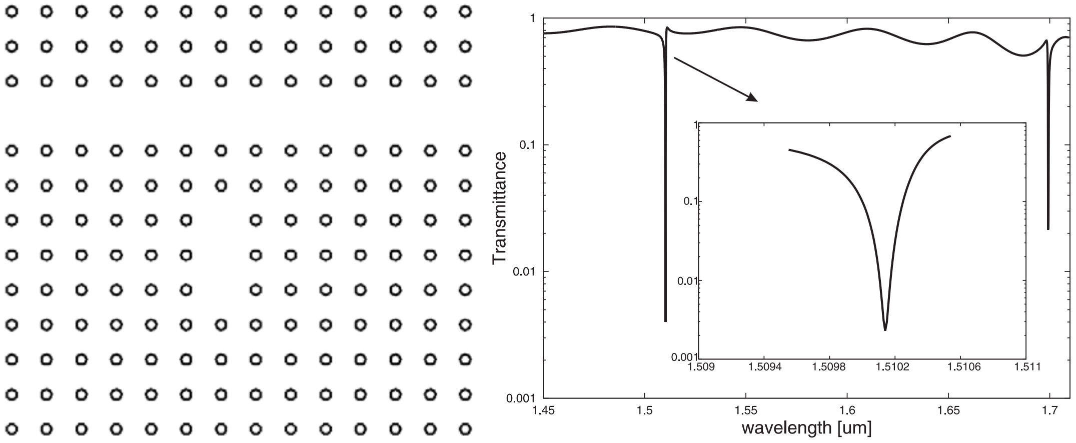

Fig. 1. Left:PC notch filter. Right: transmittance spectrum of the notch filter

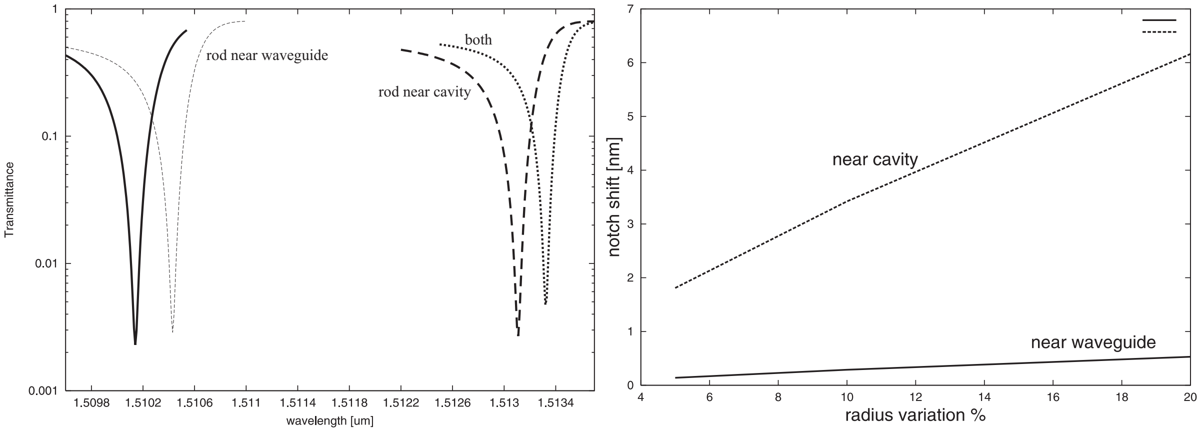

Fig. 2. Left: transmittance spectrum of the notch filter. Right: notch shift.

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