Effective and Flexible Analysis for Propagation in Time Varying Waveguides

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The equivalence between the propagation of dispersive modal fields in two and three dimensional waveguides, and plane waves in a one dimensional plasma is presented. A computationally efficient time domain integral equation is discussed for this important practical case and its stability is improved by means of both a semi-implicit formulation and the use of rectangular meshing.

Keywords: time domain analysis, numerical algorithm, dimensionality reduction.

Introduction

Recently, significant interest has been focused upon the interaction of electromagnetic signals and time varying media, [1,2]. The motivation for studying such phenomena include, wavelength shifting and terahertz wave generation applications, as well as gaining insight into the behaviour of high speed switches and lasers. In contrast to the early research in this area, which concentrated upon plane waves, the leading edge of this work is now considering the effects of time varying materials upon optical fields transversely confined by waveguide geometries.

Computational methods that can account for time varying and inhomogeneneous media, include the finite difference time domain method, FDTD, the transmission line method, TLM as well as time domain integral equation techniques. A particular advantage of the latter approach is that it can often identify the general properties of a particular class of problems rather than just solving specific instances, [3]. Unfortunately in all cases, the move from 1D plane wave problems to 2D, let alone 3D waveguide configurations severely increases the computational overhead incurred. However, for initial design purposes, it is recognised that it is not strictly necessary to model the complete 2D or 3D nature of the structure, rather it is sufficient to consider just the fundamental modal fields which indicate the presence of the waveguide confinement by possessing frequency dependent propagation constants and modal impedances. Exclusively concentrating upon the fundamental modes is justifiable given that the temporal changes in the waveguide parameters are usually small in practice so that excitation of higher order transverse modes is of secondary importance to the overall behaviour observed. The significant advantage gained by this approach is a reduction in the dimensionality of the problem that needs to be simulated.

Theory

If the transverse mode shape is frequency independent, which is a reasonable engineering approximation for semiconductor slab waveguides confined by large refractive index steps such as occur in SOI structures, then it is straightforward to demonstrate that the dispersive effects exhibited by a waveguide mode whose cutoff frequency is ω_c, are the same as those observed for plane waves in a 1D plasma defined by the following relationship between electric field and electric flux density, $D(t) = \varepsilon_o \varepsilon_1 E(t') + \int^t dt' \ (t-t')\omega_c^2 E(t')$ where ε_1 is the relative permittivity of the material filling the guide. The electric field in a 1D structure satisfies Volterra integral equation, [3],

$$E(t,z) = E_o(t,z) - \frac{1}{2\varepsilon_o \varepsilon_b} \frac{d}{dt} \int_0^t dt' \int_0^a dz' \ \delta\left(t - t' - \frac{|z-z'|}{v_b} \right) \left[P(t',z') - \varepsilon_o(\varepsilon_b - 1) E(t',z') \right] \qquad (1)$$

where ε_b is a background relative permittivity, P is the polarisation of the media, E_o is the excitation field and t and z are the time and space coordinates. It is a significant advantage of the approach, compared to FDTD for example, that the domain of the spatial integration only encompasses the discontinuity, i.e. the region whose properties differ from the background medium, here taken as $0<z<a$. Similarly, there is no need to terminate the calculation window with artificial absorbers as the kernel of the integral equation intrinsically contains the correct asymptotic behaviour at infinity. To perform the simulation of the modal fields in a 2D waveguide as introduced above, E is now interpreted as the modal amplitude and the equivalent 1D material modelling the modal dispersion is defined by the polarisation function,

$$P(t,z) = \varepsilon_o(\varepsilon_1 - 1)E(t,z) + \varepsilon_o \omega_c^2 \int_o^t dt'(t-t')E(t',z) \qquad (2)$$

In previous simulations of 1D plasmas using Volterra equations, [4], the field was discretised in space-time using a square mesh with respect to the speed of light in the background material, i.e. $\Delta z = v_b \Delta t$. Furthermore, the integration over t' in (1) was performed using a semi-open integration scheme and the derivative with respect to t evaluated using a backward difference. Given that using $\Delta z = v_b \Delta t$ is tantamount to operating at precisely the Courant condition, these features give rise to a slightly explicit scheme which causes problems for stability and accuracy. To overcome these problems, a very simple Crank-Nicolson approach is adopted; a central difference formula is used for the derivative with respect to t, coupled with $E(t - dt/2) = \alpha E(t) + (1-\alpha)E(t-dt)$. Using $\alpha=0.5$ ensures maximal accuracy and theoretical stability, but in practice α is set slightly larger than 0.5 to compensate for rounding error induced instability. As shall be shown below, this revised implementation of the algorithm, reported here for the first time, significantly enhances its utility.

A second novel and complimentary change to the algorithm is to relax the need for a square mesh in space-time, which effectively allows Δt to be lower than that of the Courant condition. Evaluating the numerical quadratures along lines of constant $z \pm vt$ which are required in (1), involves interpolation for non-square meshes, although this is straightforward to implement in practice. Here we specifically demonstrate the stability and accuracy improvements that can be obtained by using a rectangular mesh with $\Delta t = \Delta z/(2v_b)$.

Results

First, the equivalence between modal propagation in waveguide and 1D propagation in a plasma as well as the efficacy of our improved algorithm shall be shown. In figure 1, an incident pulse, with a Gaussian distribution in both time and space, strikes a section of waveguide sandwiched between 2 free space regions and frequency dependent reflection and transmission occurs. The problem parameters used here and below have been chosen to clearly demonstrate the approach and are therefore not necessarily realistically achievable.

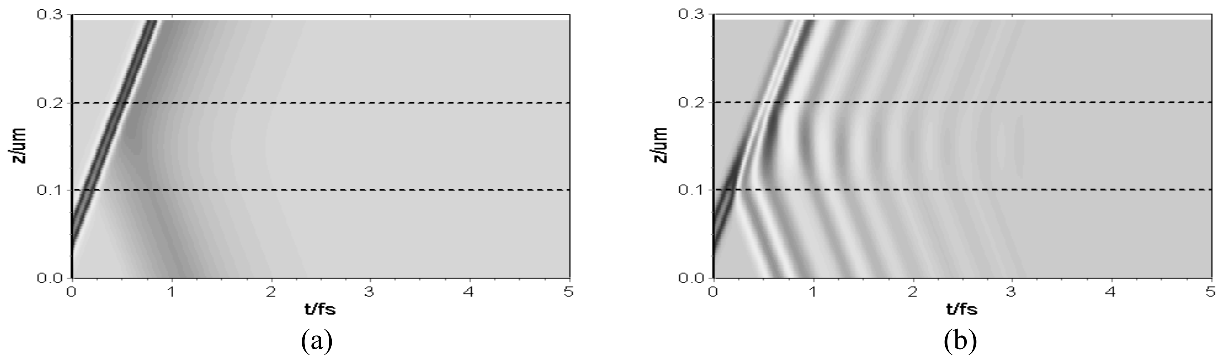


Figure 1: The modal amplitude excited by a Gaussian pulse striking a section of waveguide, $0.1\mu m<z<0.2\ \mu m$ with (a) f_c=500THz and (b) f_c=2500THz. Here, $\varepsilon_1 = \varepsilon_b = 1$, $\Delta t = \Delta z/v_b = 0.01$fs and $\alpha=0.6$.

To precisely quantify the accuracy of the numerical implementation, figure 2 compares the transmission coefficient obtained by Fourier transforming the time dependent field just before and after the waveguide with the exact theoretical result.

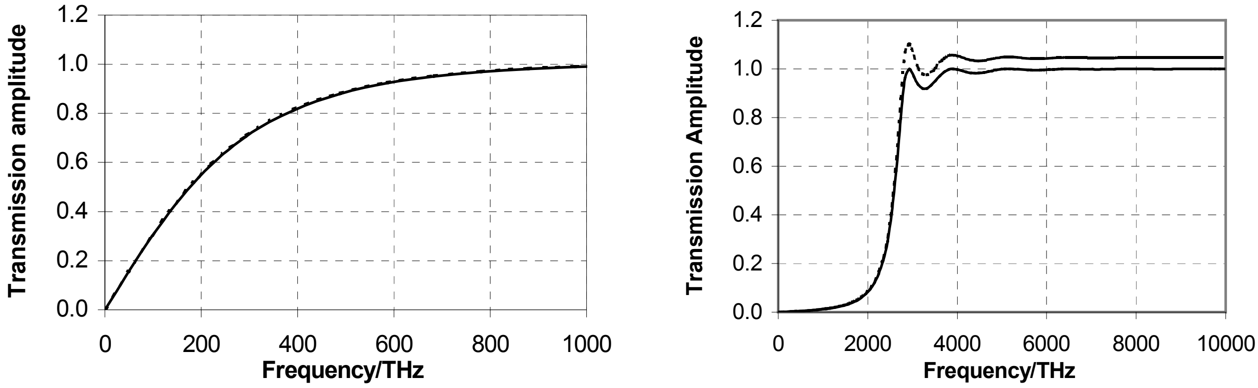


Figure 2: The numerical (dotted) and theoretical (solid) transmission coefficient obtained for the examples of figure 1

Although the agreement is excellent for the case of relatively weak dispersion, for the case of more severe dispersion there is both an overshoot at the cutoff frequency as well as a "steady state" pass band amplitude error. This is attributable to the use of a stability factor of α=0.6 and a fairly large time step. To illustrate this, the pass band error is plotted in figure 3(a) against α for different Δt.

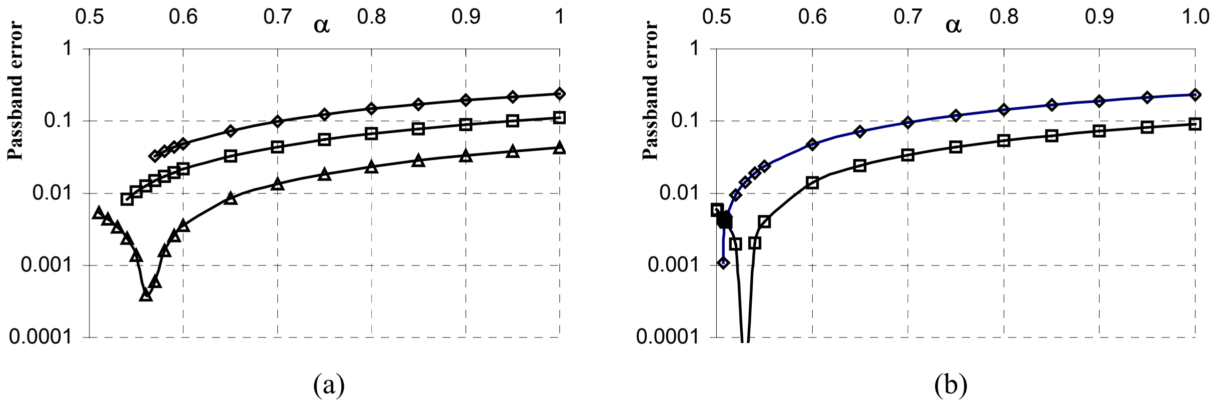


Figure 3: The effect of α and Δt on the errors for a Square (a) and Rectangular (b) mesh for the example of figure 1b: (a) diamonds ($\Delta t=\Delta z/v_b$ =0.01fs), squares ($\Delta t=\Delta z/v_b$=0.005fs) and triangles ($\Delta t=\Delta z/v_b$=0.0025fs); (b) diamonds ($\Delta t=\Delta z/v_b/2$=0.005fs) and squares ($\Delta t=\Delta z/v_b/2$=0.0025fs).

It is clear that this measure of error is significantly reduced as both α approaches 0.5 and by using a smaller time step. However, it is also observed that each of the curves starts at a value α which is greater than 0.5, as below this value instability occurs. Therefore it is not possible to obtain the accuracy that would appear asymptotically available for a given value of Δt. As using smaller Δt implies using smaller Δz in a square mesh, improving the accuracy this way incurs an undesirable increase in numerical effort. This is precisely the reason for considering rectangular meshing as it moves the algorithm away from the accuracy-stability knife-edge associated with the Courant condition. Figure 3(b) shows the same measure of error obtained from a mesh with $\Delta t=\Delta z/(2v_{b)}$.

As a further visual illustration of the wavguide-plasma equivalence we consider the case of a rib waveguide resonator. A section of waveguide is sandwiched between two sections of waveguide having a higher cutoff frequency, and this combination is surrounded by free space. Modelling each waveguide as an appropriate 1D plasma (without consideration of the mismatch in modal profiles) is in essence a time domain effective index model, the frequency domain a of which has achieved widespread acceptance as a fast initial design tool. Figure 4 shows the fields excited by a Gaussian pulse incident from free space and the frequency dependent confinement and resonances are plainly seen.

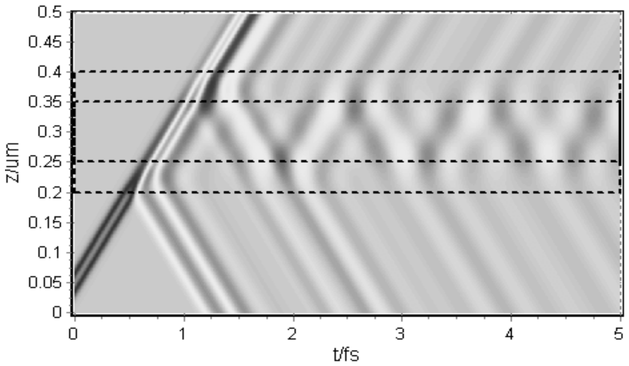
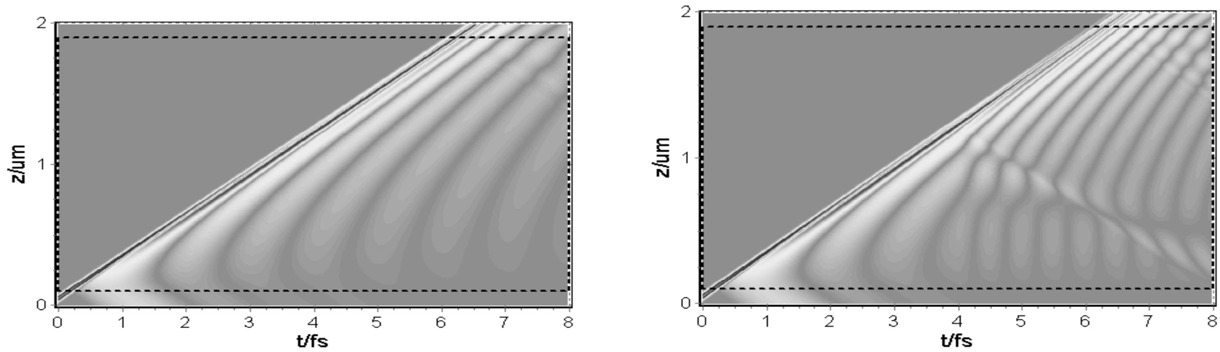


Figure 4: The modal amplitude excited by a Gaussian pulse striking a cascade of three waveguides with f_c=2500THz for 0.2μm<z<0.25μm and 0.35μm<z<0.40μm and f_c=1000THz for 0.25μm<z<0.35. $\Delta t = dz/v_b$=0.005μm, α=0.6 and $\varepsilon_1 = \varepsilon_b$=1.

Finally, attention is focused on the case of a time jump in the waveguide parameters. Figure 5 shows the effect of switching the cutoff frequency of the waveguide region as the field passes through it, the reflection at the instant of switching, t=4fs, being clearly visible.



(a) (b)

Figure 5: The modal amplitude excited by a Gaussian pulse striking a section of waveguide, 0.1μm<z<1.9 μm with f_c=500 THz, $\varepsilon_1 = \varepsilon_b$=1, $\Delta t = \Delta z/v_b$=0.01fs and α=0.6. In (b) f_c is switched to 2500THz at t=4fs.

Conclusion

In this paper we have presented and demonstrated two practical improvements for time domain simulations. Firstly, that simulations of 2D and 3D waveguide structures can be efficiently performed using a time domain effective index approach, which promises fast initial design tools. Secondly, for Volterra integral equation formulations, both the use of stabilised algorithms and non-square meshes have been quantifiably shown to substantially improve the accuracy that can be obtained. Finally, qualitative examples of dealing with more complex geometries as well as time variant cases have been presented to illustrate the flexibility of the approach.

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