Detailed analysis of spatiotemporal stability of the ultra-short optical pulses propagating in non-linear step-index optical waveguide

Elena A.Romanova, Leonid A.Melnikov
Saratov State University, Astrakhanskaya 87, 410026 Saratov, Russia
romanova@optics.sgu.ru

Propagation of the ultra-short optical pulse through a boundary of linear and non-linear segments of planar dielectric waveguide with step-like distribution of the refractive index is simulated numerically. Impact of self-steepening and normal dispersion effects as well as role of retarded non-linear response of the waveguide material are analysed separately in detail. Solution of the paraxial wave equation with non-linear susceptibility is shown to be stable in space in time provided that the quasi-static approximation is used.

Keywords: optical waveguides, ultra-short pulses, Kerr-like non-linearity, spatiotemporal dynamics

Recent experiments with propagation of femtosecond optical pulses in fused silica has demonstrated a complicated dynamics of both spatial and temporal pulse distribution that reveal itself in splitting of the pulse and filamentation of the gaussian beam [1]. These effects are mainly due to the high peak intensities of the beam focused on a bulk silica with a relatively small Kerr-constant $n_2 \approx 10^{-16} cm^2/W$. In optical waveguides, self-action of ultra-short pulses was considered before predominantly as a temporal self-action but the self-focusing effect was assumed to be negligibly small [2]. However the above mentioned experiments with fused silica as well as design of new glasses with greater non-linearity [3] show that there is a need to consider both the spatial and temporal effects of high-intensity pulse propagation through dielectric waveguiding structures.

In our previous works, we studied spatial transient process of fundamental mode propagation through a boundary between linear and non-linear segments of step-index waveguide [4]. Behind a region of spatial transient process in the non-linear segment, a steady-state field distribution was obtained as a numerical solution of paraxial wave equation with non-linear susceptibility (so-called "non-linear mode"). This effect depends on geometry of the guiding structure and can be used to manage parameters of the propagating radiation. However these CW results can be generalised onto the pulsed power propagation provided that the pulse duration is much greater than period of optical oscillations and non-linear response of the waveguide material.

In this work, we simulate numerically propagation of femtosecond optical pulse in planar step-index waveguide with the Kerr-like non-linearity embedded and analyse spatiotemporal dynamics of slowly varying envelope $A(x,z,t)$ of the total filed $E(x,z,t) = A(x,z,t)exp(i\omega t - i\beta z)$ in the quasi-static approximation and with account of self-steepening effect and finite non-linear response as well as second-order dispersion of the waveguide material.

Numerical modeling is based on solution of the (2+1) paraxial wave equation written in a moving-frame coordinate system ($z = z,\ \ t = t_0 - z/u$) :

$$2i\beta\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial x^2}+\varepsilon_1\frac{\partial^2 A}{\partial t^2}+(k^2n^2(x)-\beta^2)A+2i\varepsilon_2\frac{\partial(|A|^2\ A)}{\partial t}+\varepsilon_3\left(|A|^2\ A-\tau_R\frac{\partial|A|^2}{\partial t}A\right)=0 \qquad (1)$$

where $\varepsilon_1 = k_2k/\tau^2(0)$, $\varepsilon_2 = n_2k/(c\tau(0))$, $\varepsilon_3 = n_2k^2$, k_2 is dispersion coefficient, n_2 is the Kerr constant, k is the mean wave number, c is the light velocity in vacuum, β is longitudinal propagation constant of the TE_0 mode, τ_R is the retarded non-linear response, $\tau(0)$ is initial pulse duration (time coordinate t and τ_R are normalised to $\tau(0)$). The waveguide is assumed to have step-index distribution of the refractive index: $n(x) = n_{co}$ ($|x| \le d$), n_{cl} ($|x| > d$), where d denotes half-width of the waveguide core (in our simulations we used n_{co}.= $\mathbf{1.491}$, $\mathbf{n_{cl} = 1.487}$, $\mathbf{d=3\mu m}$). The alternative-direction implicit method was used to solve the six-point finite-difference equations obtained from Eq.(1) with improved interface conditions [5]. Non-linear part of the dielectric permittivity was inserted into the finite-difference scheme in such a way that the value of *A(x,z,t)* obtained on a previous layer was used as a zero approximation for the next layer. In order to avoid the boundary reflection problem, complex scaling of coordinates was applied near the grid edges [6] beginning from $x=X_b=15d$. Initial field distribution $A(x,0,t)=A_0\psi(x)\exp(-t^2/2)$ with transverse profile $\psi(x)$ of linear TE_0 mode (Fig.1,a) was launched numerically into a non-linear waveguide segment where a spatial transient process was observed (Fig.1,b).

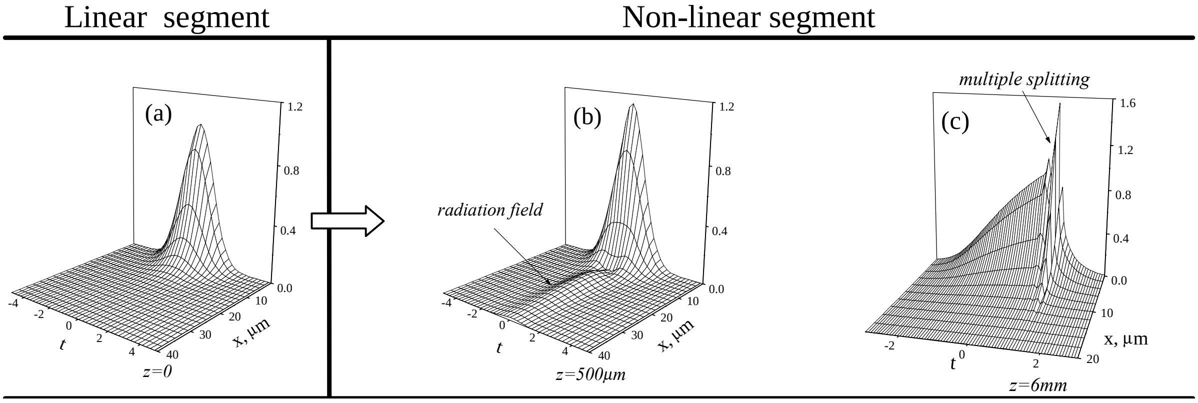

Fig.1 Spatiotemporal distribution of the pulse intensity

In the quasi-static approximation ($\varepsilon_1=0,\ \varepsilon_2=0,\ \tau_R=0$), we assume that only self-focusing and temporal self-action determine non-linear dynamics of the total field so that its spatial and temporal distribution depends on input power and doesn't depend on the initial pulse duration (Fig.2). Similarly to CW propagation [4], total losses resulting from the leakage of radiation field outside the guiding region grow with input power (in our simulations we used the output parameter $\mathrm{T}(X,z)=\int_{-X}^{X} dxdt\,|A(x,z,t)|^2 \,/\, \int_{-X}^{X} dxdt\,|A(x,0,t)|^2$).

Behind the region of spatial transient process, stable spartiotemporal field distribution is observed with increased power flow propagating inside the core ($T(d,z)>1$). Pulse duration on the waveguide axis $\tau_a(z)$ is less than its initial value ($\tau_a(z)/\tau(0)<1$) and increases with transverse coordinate tending to $\tau(0)$ in the cladding. The beam width also is minimal at the pulse peak. In general, the stable field distribution is consistent with "non-linear mode" or "spatial soliton" representation.

Self-steepening effect and retarded non-linear response are the factors which destroy stability of the solution so that constant leakage of the power outside the guiding region is observed along the propagation distance (Fig.3).

The self-steepening effect results from intensity dependence of the group velocity and manifests itself through a shift of the pulse peak and optical shock in the trailing edge of the pulse (Fig.4). This variation of the temporal (longitudinal) pulse distribution influences the transverse profile and cancels balance between self-focusing and diffraction so that stable distribution is no longer obtained.

The retarded non-linear response similarly shifts the pulse peak to the trailing edge however don't disturb symmetry of the frontal and trailing parts. With $\tau_R=0.1$ that is typical for femtosecond pulse propagation at $1.53\mu m$ in fused silica, this effect is much less than the self-steepening one.

In the far-field, mutual action of the self-steepening and retarded non-linear response results in the pulse splitting (Fig1,c). For increasing input power or decreasing pulse duration, the splitting moves to the near field.

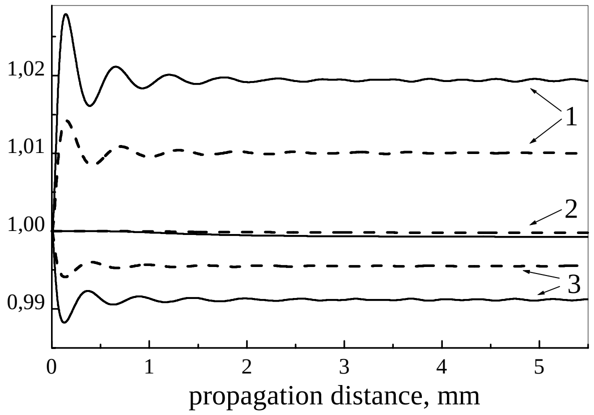

Fig.2 Quasi-static approximation: 1 - $T(d,z)$; 2 - $T(X_b,z)$; 3- $\tau_a(z)/\tau(0)$; solid curves – $n_2I=0.002$; dashed curves - $n_2I=0.001$.

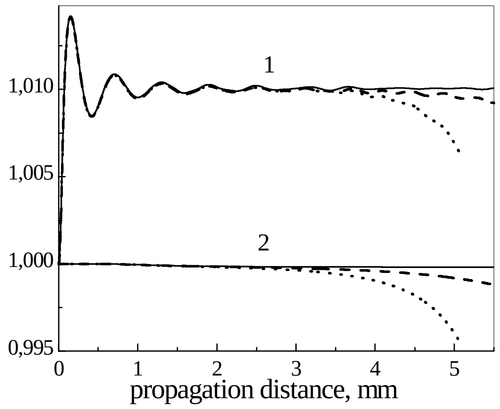

Fig.3 Self-steepening effect: 1 – $T(d,z)$; 2 – $T(X_b,z)$; $n_2I=0.001$; dashed curves- $\tau(0)=30fs$, dotted ciurves - $\tau(0)=20fs$.

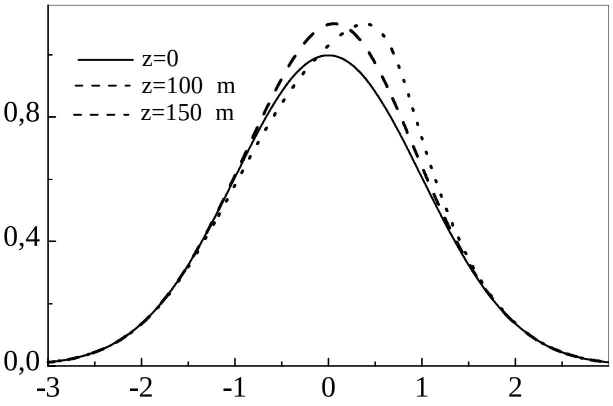

Fig.4 Displacement of the pulse peak due to the self-steepening effect. $n_2I=0.001$, $\tau(0)=20fs$

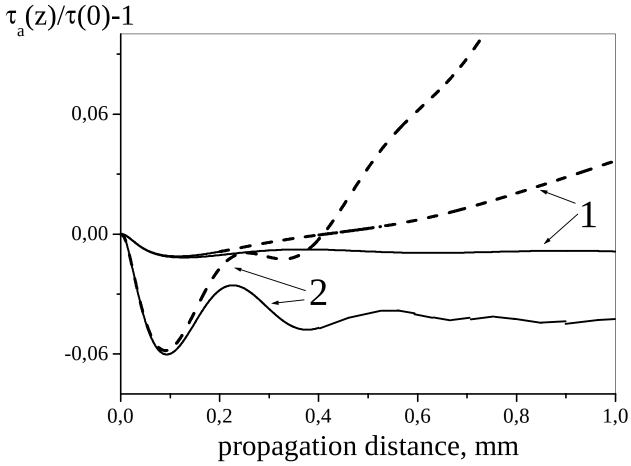

Fig.5 Relative variation of the pulse duration with account of normal dispersion (dashed curves) and in quasi-static approximation (solid curves):

1 - $n_2I=0.002$, $\tau(0)=30\,fs$, $k_2 = -0.001\,fs^2/\mu m$;

2 - $n_2I=0.01$, $\tau(0)=10\,fs$, $k_2 = -\,0.036\,fs^2/\mu m$.

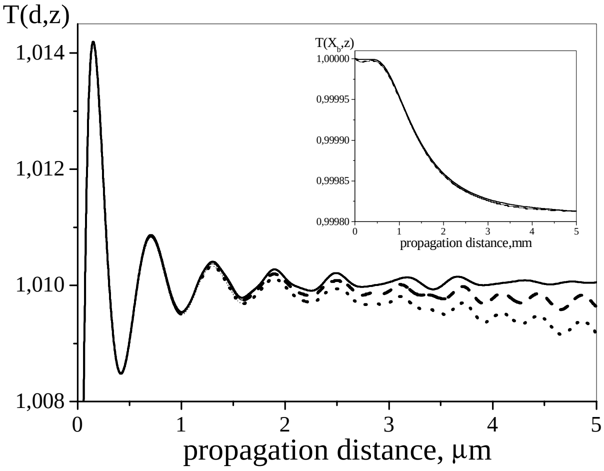

Fig.6 Effect of normal dispersion: dashed curves - $\tau(0)=30fs$, dotted curves - $\tau(0)= 10\,fs$; $n_2I=0.002$, $k_2 = -\,0.036\,fs^2/\mu m$.

Normal group velocity dispersion ($k_2<0$) plays an important part in propagation of ultra-short pulses. In idealized model based on non-linear Schrodinger equation, the self-phase modulation and the second-order dispersion are accounted for. As a result of balance between non-linear and dispersion effects, this model gives a stable solution (soliton) for space-time field distribution [2]. If diffraction term is taken into consideration, the normal group velocity dispersion prevents catastrophic collapse of high-power beam in a uniform Kerr medium via multi-splitting of the pulse, however the self-focusing and splitting events are spatially separated [1]**.** In the step-index waveguide, character of spatiotemporal dynamics of the total field depends also on magnitudes of the dispersion and diffraction lengths: $l_{ds}=\tau^2(0)k/(k_2\beta)$ and $l_{df}=\beta d^2/2$. If $l_{ds} >> l_{df}$ that is typical for picosecond pulse propagation in optical waveguide, the pulse broadening is small over the region of spatial transient process initiated by the self-focusing effect (Fig.5,curves 1). In this case, the self-focusing and the dispersion broadening are separated in space and can be treated independently. If a femtosecond pulse is taken into consideration, the characteristic lengths can be of the same order, $l_{ds} \sim l_{df}$, so that significant variations of the pulse duration along the propagation distance are observed (Fig.5,curves 2). These variations of the pulse duration change in turn the spatial distribution of the field leading to constant leakage of the power outside guiding region along the propagation distance (Fig.6). Contrary to the self-steepening effect, the frontal and trailing parts of the pulse keep symmetric. Comparison of the parameter $T(X_b,z)$ (Fig.3 and Fig.6) shows that the radiation field excited as a result of dispersion effect is more paraxial than the radiation field leaking from the guiding region due to the self-steepening effect.

In summary, simulations of the modal field transformation behind the boundary of linear and non-linear segments of planar waveguide has demonstrated that the self-steepening effect, retarded non-linear response of the waveguide material and normal material dispersion create instability in the high-power ultra-short pulse spatiotemporal dynamics. This results in constant leakage of the power outside the guiding region. The self-steepening effect leads also to multiple pulse splitting in the far-field.

This results can be generalized on any irregular guiding structures of integrated circuits where spatial and temporal transformation of high-intensity pulsed power takes place.

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