Time Domain Numerical Model for linear and nonlinear Grating-Assisted Codirectional Coupler

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A time domain model is presented for the dynamic analysis of linear and nonlinear devices based on coupled modes propagating with different group velocities. As a test, the numerical algorithm is applied to the analysis of a linear grating assisted co-directional coupler and the limitation of the method are investigated.

Keywords: linear and nonlinear co-directional coupling, GACC filter, TDTW model, simulation

Introduction

Narrow band filters, tunable over a wide wavelength range and integrated with other photonic devices, are important elements for WDM multi/demultiplexing systems. Apart from the distributed Bragg reflectors based on contra-directional coupling, linear and nonlinear filters having a Grating Assisted Codirectional Coupler (GACC) [1] have also been used in many laser and integrated optic structures [2]-[5]. Fig. 1(a) show the typical GACC structure with the two waveguides coupled through a long period grating with pitch Λ_{coupl}. As shown in Fig. 1(b) the modes of the two waveguides, coupled through the perturbation created by the grating, propagate co-directionally with different propagation constants, $\beta_1(\omega)$ and $\beta_2(\omega)$. The different dispersion of the two propagation constants gives different group velocities, v_{g1} and v_{g2}, and the group velocity difference determines the GACC filter bandwidth and tuning capability [6]. Several papers have been published in the literature on the spectral domain analysis and design of these filters [5]-[7], but, to our knowledge, there is not any model that analyze dynamically the propagation of two coupled modes having different group velocities. Furthermore this kind of simulation is essential whenever waveguide nonlinear effects have to be included [5]. The purpose of this paper is to present an extension of the Time Domain Travelling Wave (TDTW) model [8] applied to this case. The TDTW equations and the numerical algorithm for their solution presented in the literature concern indeed only with the case of two coupled modes travelling with the same group velocity, v_g, as for example in DFB and DBR lasers [8],[9]. For these devices the TDTW equations are numerically solved using a space-time discretization grid, defined to satisfy the condition $\Delta z = v_g \Delta t$, where Δz and Δt are respectively the space and time discretization steps. This condition allows the field samples to propagate according to the differential equation characteristic lines, keeping null the numerical error accumulated during the propagation [9]. We have observed that the the standard TDTW approach [8],[9] can not take into account the case of modes with different group velocities and this leads, for example, to significant errors in the simulated GACC filter bandwidth and tunability. A proper numerical method and the conditions to solve correctly the TDTW system of coupled mode equations with different group velocities is thus presented. The numerical errors that result moving away from these proper working conditions are also evaluated.

Model and numerical algorithm

The GACC TDTW equations can be obtained from the spectral domain equations that result from coupled mode theory [1]. To define the notations we report the final equations for

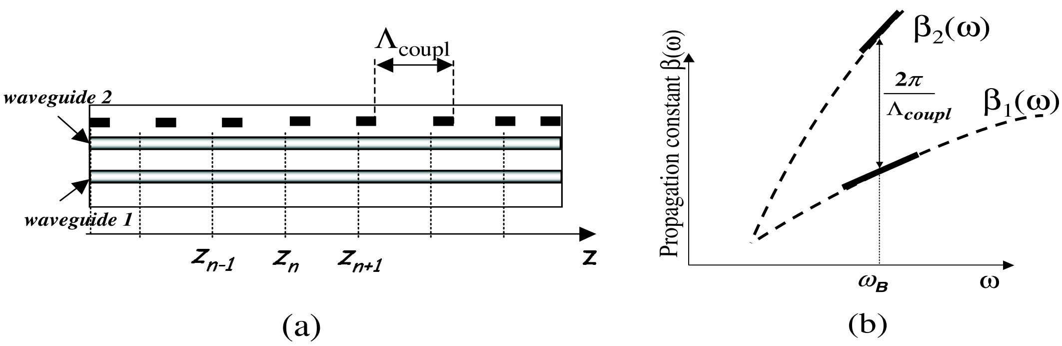

Fig.1. (a) GACC structure with the spatial discretization grid and (b) scheme of the dispersion of the propagation constants.

the slow varying forward components, $\mathcal{A}_1^+(z,t)$ and $\mathcal{A}_2^+(z,t)$, of the electric field.

$$\begin{cases} \dfrac{\partial \mathcal{A}_1^+(z,t)}{\partial z} + \dfrac{1}{v_{g1}}\dfrac{\partial \mathcal{A}_1^+(z,t)}{\partial t} = -j\,\tilde{\delta}_1\,\mathcal{A}_1^+(z,t) + j\,k\,\mathcal{A}_2^+(z,t) \\ \dfrac{\partial \mathcal{A}_2^+(z,t)}{\partial z} + \dfrac{1}{v_{g2}}\dfrac{\partial \mathcal{A}_2^+(z,t)}{\partial t} = -j\,\tilde{\delta}_2\,\mathcal{A}_2^+(z,t) + jk\,\mathcal{A}_1^+(z,t) \end{cases} \tag{1}$$

These slow varying components have been obtained for each waveguide, i, extracting from the time domain electric field an arbitrary reference pulsation, ω_0, and a reference propagation constant $\beta_{i0} = \omega_0/c \cdot n_{eff\,i0}$, with c the light velocity and $n_{eff\,i0}$ the effective refractive index of waveguide i. In equation (1) k is the mode coupling coefficient and $\tilde{\delta}_1$ and $\tilde{\delta}_2$ are in general complex detuning terms that take into account the detuning of the reference pulsation ω_0 from the Bragg condition, the waveguide loss, gain and nonlinear effects. Analogous equations can be also obtained for the backward components. For the numerical solution the coupler structure has been divided in slices $\Delta z = v_{g_{num}}\,\Delta t$ (Fig. 1a), having chosen $v_{g_{num}} = \max\{v_{g\,1}, v_{g\,2}\}$ for convergency reasons [10]. In the following we will assume that $v_{g\,1}$ is the maximum group velocity. The system of equations (1) has then been solved with a split step algorithm. For each spatial node z_n, we have calculated the field propagation from time instant t_m to t_{m+1} in two steps. In the first step we have solved the temporal propagation equation, obtaining, as shown in equations (2) and (3), the field samples $\overline{\mathcal{A}}^+_{1\,n,m+1}$ and $\overline{\mathcal{A}}^+_{2\,n,m+1}$

$$\frac{\partial \mathcal{A}_1^+(z,t)}{\partial z} + \frac{1}{v_{g\,1}}\frac{\partial \mathcal{A}_1^+(z,t)}{\partial t} = 0 \implies \overline{\mathcal{A}}^+_{1\,n,m+1} = \mathcal{A}^+_{1\,n-1,m} \tag{2}$$

$$\frac{\partial \mathcal{A}_2^+(z,t)}{\partial z} + \frac{1}{v_{g\,2}}\frac{\partial \mathcal{A}_2^+(z,t)}{\partial t} = 0 \implies$$
$$\overline{\mathcal{A}}^+_{2\,n,m+1} = \mathcal{A}^+_{2\,n,m} - \frac{v_{g\,2}\Delta t}{2\,\Delta z}[\mathcal{A}^+_{2\,n+1,m} - \mathcal{A}^+_{2\,n-1,m}] + \frac{1}{2}(\frac{v_{g\,2}\,\Delta t}{\Delta z})^2[\mathcal{A}^+_{2\,n+1,m} - 2\mathcal{A}^+_{2\,n,m} + \mathcal{A}^+_{2\,n-1,m}] \tag{3}$$

Since $\Delta z = v_{g_1}\,\Delta t$, equation (2) is solved exactly [9] whereas equation (3) have been solved numerically using a second order Lax-Wendroff scheme [10]. In the second step we have then included the coupling and the complex detuning, $\tilde{\delta}_1$ and $\tilde{\delta}_2$, solving the following equations:

$$\frac{\mathrm{d}\,\mathcal{A}_1^+}{\mathrm{d}\,z} = -j\,\tilde{\delta}_1\,\mathcal{A}_1^+(z) + jk\,\mathcal{A}_2^+(z) \quad \text{and} \quad \frac{\mathrm{d}\,\mathcal{A}_2^+}{\mathrm{d}\,z} = -j\,\tilde{\delta}_2\,\mathcal{A}_2^+(z) + jk\,\mathcal{A}_1^+(z) \tag{4}$$

The transfer matrix for the solution of (4) can be easily obtained from the calculation of eigenvalues and eigenvectors of (4). Thus we obtain:

$$\begin{pmatrix} \mathcal{A}^{+}_{1\,n,m+1} \\ \mathcal{A}^{+}_{2\,n,m+1} \end{pmatrix} = \begin{pmatrix} T_{11}(\Delta z) & T_{12}(\Delta z) \\ T_{21}(\Delta z_2) & T_{22}(\Delta z_2) \end{pmatrix} \begin{pmatrix} \overline{\mathcal{A}}^{+}_{1\,n,m+1} \\ \overline{\mathcal{A}}^{+}_{2\,n,m+1} \end{pmatrix} \tag{5}$$

We have observed that evaluating the matrix elements T_{11}, T_{12} in $\Delta z = v_{g\,1}\,\Delta t$ and T_{21}, T_{22} in $\Delta z_2 = v_{g\,2}\,\Delta t$, the solution obtained is less affected by numerical errors respect to the solution calculated evaluating all the elements in the same $\Delta z = v_{g\,1}\,\Delta t$, as done in a standard split-step TDTW approach [9].

Numerical results

To demonstrate the accuracy and efficiency of the proposed time-domain method, we analyze a 540 μm long GACC, with a grating coupling coefficient of 29 cm^{-1} and group refractive index $n_{g1} = 3.524$ and $n_{g2} = 3.867$. We show in Fig.2a and 2b the power distribution of the CW fields $\mathcal{A}^{+}_{1}(z,t)$ and $\mathcal{A}^{+}_{2}(z,t)$ when the GACC Bragg pulsation is equal to the reference pulsation ω_0. The boundary conditions are $\mathcal{A}^{+}_{1}(0,t) = 1$ and $\mathcal{A}^{+}_{2}(0,t) = 0$. Fig. 2a shows that, evaluating all the transmission matrix elements in the same $\Delta z = v_{g\,1}\,\Delta t$ the solution is not correct because the upper waveguide receives too much power from the lower one and the total power is not conserved. On the contrary, as demonstrated in Fig. 2b, the total power maintains constant along z, if the transfer matrix is calculated as indicated in (5). To test the method also in a

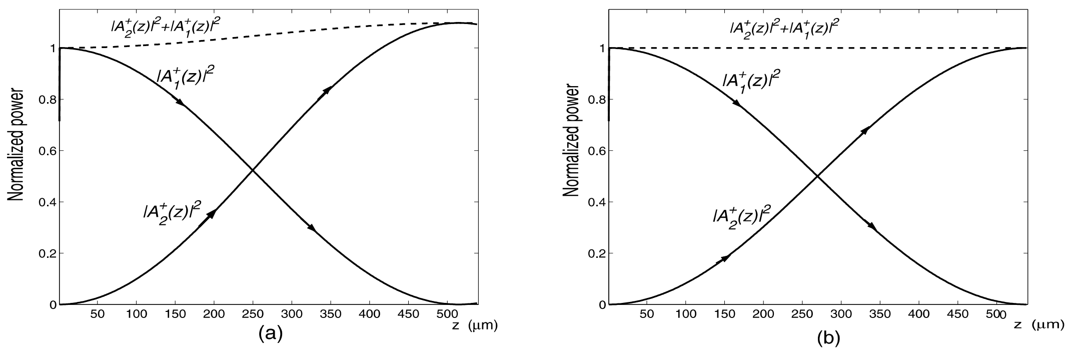

Fig. 2. Normalized power distribution, along the GACC, of the CW forward fields at the Bragg wavelength. Simulation evaluating (a) all the transfer matrix elements in $\Delta z = v_{g1}\Delta t$ and (b) as indicated in equation (5).

wide wavelength range, we compare in Fig. 3a-b the GACC transmission spectrum, obtained with our TDTW analysis, with the exact result obtained with a spectral domain analysis. Fig. 3a shows that in case of zero detuning, $\Delta\lambda$, between the numerical procedure extracted reference wavelength (λ_0=1570 nm) and the GACC Bragg wavelength, λ_B, the two solutions practically overlap. However the error becomes significant when we move away from $\lambda_B = \lambda_0$, as shown in Fig. 3b for $\Delta\lambda = 20$ nm. This leads to errors in the simulated GACC Bragg wavelength and maximum transmission coefficient. We have observed that these numerical errors are mainly due to the dispersion and distortion introduced by the Lax-Wendroff scheme [10] and can be partially compensated at λ_B since they can be analytically evaluated [10] for each wavelength component of the fields. Fig. 3c-d report, as an example, the errors on GACC Bragg wavelength and peak transmissivity versus the detuning $\Delta\lambda$ when the numerical algorithm is applied without and with the application of a simple error correction scheme at λ_B.

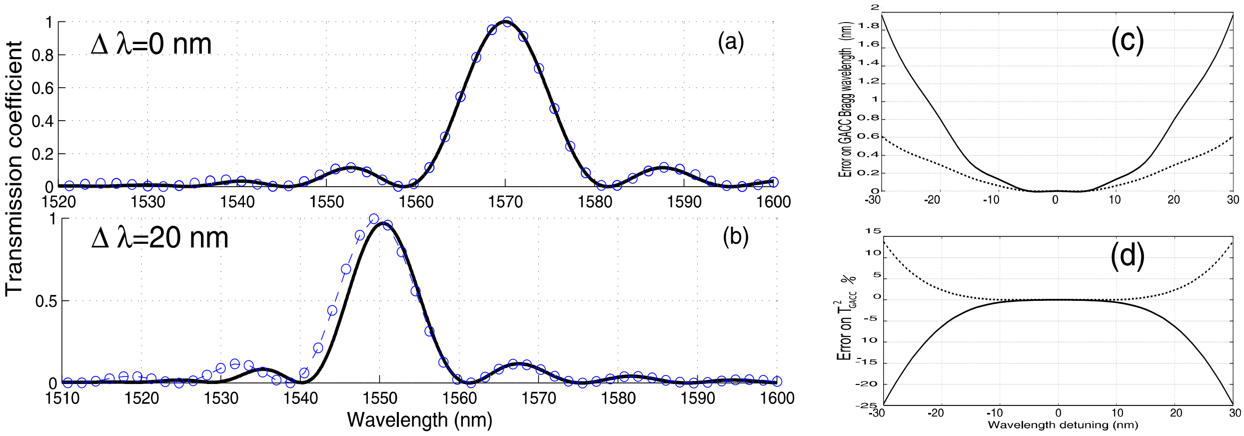

Fig. 3. GACC transmission coefficient versus wavelength for the cases we choose (a) zero and (b) 20 nm detuning of the extracted λ_0=1570 nm from the Bragg wavelength. Solid line: TDTW solution, Circle: exact reference solution. (c) Error in the GACC Bragg wavelength and (d) maximum transmissivity versus wavelength detuning without (solid line) and with (dash line) error correction.

Conclusion

We have presented a numerical method for the solution of the TDTW equations of two co-directionally coupled modes travelling at different group velocities. As a reference case to validate the proposed algorithm, we have considered a GACC filter. It has been shown that with a proper choice of the reference pulsation the numerical error is practically null over a wide wavelength range. Furthermore, for non zero detuning in the device selected reference pulsation, it can be partially compensated around $\pm$10 nm with a simple error correction procedure. The model proposed can find interesting applications in the time-domain analysis of photonic integrated devices that include a GACC such as active and nonlinear filters, lasers and whenever the nonlinearity does not allow a straightforward application of the spectral domain approach.

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